

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

THE GRADUATE COLLEGE

21 December 1973

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SUPERVISION BY HARRY PETER PLANCHON, JR.

ENTITLED THE FLUCTUATING STATIC PRESSURE FIELD IN A ROUND

JET TURBULENT MIXING REGION

BE ACCEPTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR
THE DEGREE OF DOCTOR OF PHILOSOPHY

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THE FLUCTUATING STATIC PRESSURE FIELD
IN A ROUND JET TURBULENT
MIXING REGION

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THESIS

Submitted in partial fulfillment of the requirements
for the degree of Doctor of Philosophy in Nuclear Engineering
in the Graduate College of the
University of Illinois at Urbana-Champaign

Urbana, Illinois

THE FLUCTUATING STATIC PRESSURE FIELD
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Nuclear Engineering Program
University of Illinois, 1974

An experimental and analytical study of the static pressure field in a turbulent mixing region has been conducted. The study centered around the incompressible isothermal, driven mixing region provided by a 2-1/2 in. diameter round jet. The study involved the following related parts: (1) Review and application of pressure theory, (2) Refinement and verification of the pressure transducer, and (3) An experimental determination of the static pressure field structure, with interpretation based on velocity field measurements.

The theory review shows the pressure field is composed of terms from turbulent-turbulent interactions and turbulent-mean shear interactions. The turbulent-mean shear term in a mixing layer is clearly shown to depend on transverse turbulent velocity, mean velocity gradient, and a length scale. The theory is extended, with the development of a definitive velocity field model, so that pressure intensities are predicted. The predicted pressure intensity distributions agree well with experimental results.

Measurements of the fluctuating velocity and static pressure fields were made with x-configuration hot wire sensors and a recently developed miniature pressure transducer, respectively. The pressure transducer, a refined model of the instrument developed by Spencer, was shown to be smaller (0.060 in. diameter) than the turbulent velocity or pressure

scale, respond linearly to pressure, have a flat frequency response to 10 kHz, have adequate sensitivity for inflow measurements, and have a small direct response to the fluctuating velocity field. A detailed model with experimental verification showed that the error from fluctuating velocity was less than 5%.

Both single point and two point measurements of the pressure and velocity field were made. It was found from the pressure spectra and space correlations that the pressure structure is large and slowly varying. The axial space correlation was found to have a damped periodic structure. The discrete wavelength of this structure increases with downstream distance and compares well with peaks in the pressure and velocity spectra. The convection velocity of the pressure structure was found to be nearly constant throughout the mixing layer. The convected frame time scale was found to be long--almost two times as large as the velocity time scale.

A separability model of the space correlations was shown to accurately represent the 3-dimensional space correlation in terms of three independent correlation measurements.

ACKNOWLEDGEMENT

With great appreciation, the author acknowledges his advisor's interest and guidance during the whole course of graduate study. Dr. B. G. Jones' continuing efforts have provided the rare, if not unique, opportunity to learn and pursue a thesis investigation in a mutually supportive research group.

Dr. Bruce Spencer provided valuable initial encouragement and an appreciation for experimental methods. The jet flow facility and pressure transducers, which he designed, were foundations of the experiment.

The author's colleagues, in Dr. Jones' fluid research group, significantly influenced the course of the investigation. Testing of the pressure transducer with Robert Hammersley lead to improved design and understanding of its behavior. Early investigations with Hammersley developed the experimental procedures used during the dissertation research. David Weber's insistence that pressure intensities could be predicted and his assistance and encouragement during the work in Chapter 2 are greatly appreciated. Appendix I is part of a continuing joint effort with Weber to apply the concepts of axisymmetry to free shear flows. The dialogue with, and assistance of, Neil Howard and Dr. Charles Meek were most helpful in developing turbulent flow concepts and experimental techniques. Thanks for the experimental and data reduction assistance must go to Rodney Krich, John Brtis, and James Tomkins.

The suggestions and support from the Nuclear Engineering machine shop and office staff are greatly appreciated.

This work was supported by a NASA grant (NASA NGR-14-005-149) through the Lewis Research Center. Their financial and technical support, particu-

larly through discussion with Mr. Robert Dorsch and Mr. Allen Karchmer, is gratefully acknowledged.

A special thank you goes to the author's wife and family. Their interest and perspective made graduate study an adventure.

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NOMENCLATURE

A	= pressure transducer pressure coefficient for axial velocity
B	= pressure transducer pressure coefficient for normal velocity
b	= a measure of mixing layer width
C_1, C_2, C_3	= turbulence structural factors contributing to PT error
D	= jet diameter
E	= instrument output voltage
$F(f)$	= one-dimensional spectral function
F	= flatness
$f(\xi)$	= isotropic defining scalar
f	= frequency
G	= electronic gain
I	= intensity
$J()$	= Kraichnan's trubulent-mean shear anisotropy factor
$K()$	= Kraichnan's trubulent-turbulent anisotropy factor
K_N	= $(\overline{V_N^2}/\overline{V_A^2})^{1/2}$, velocity intensity anisotropy factor
k	= $(\overline{V^2}/\overline{U^2})^{1/2}$, velocity intensity anisotropy factor
k_1	= $(\overline{W^2}/\overline{U^2})^{1/2}$, velocity intensity anisotropy factor
L	= integral length scale
l/d	= hot-wire length to diameter ratio
m	= lateral pressure space correlation decay constant
n	= radial pressure space correlation decay constant
p	= pressure
P_{TT}^2	= mean square pressure from turbulent-turbulent interactions

P_{TM}^2	= mean square pressure from turbulent-mean shear interactions
R	= jet radius
$R_{u_i u_j}$	= correlation coefficient between u_i and u_j
r	= radial direction
u	= velocity ratio
S	= skewness
S_p	= sensitivity to pressure
Δ_i	= an empirical scale anisotropy factor for transverse velocity
T	= convected frame time scale
t	= time
u_i	= instantaneous velocity in i direction
u_i'	= fluctuating velocity in i direction
$\overline{u_i}$	= mean velocity in axial direction
u, v, w	= fluctuating velocity in axial, radial and transverse directions
u_a, u_b	= primary and secondary free stream velocities in a two-stream mixing layer
$u_A u_N$	= velocity axial and normal to the pressure transducer tip
$u_c^{u_i}$	= convection velocity of u_i
u_j	= jet exit velocity
x_i, y_i	= Cartesian tensor dimension for i th direction
$\vec{x}, \vec{y}$	= position vectors
x, r, z	= axial, radial, and lateral positions
x_0	= mixing layer virtual origin

Greek Symbols

α	$= L_x/L_y = u'/v'$, Kraichnan's anisotropy factor
β	= angle of principal turbulent shear stress
δ	= boundary layer thickness
δ^*	= boundary layer displacement thickness
ϵ_z	= offset distance of sensors in the lateral direction
$\vec{r}$	= separation vector for two point correlations
η	$= (r-R)/(x-x_0)$ similarity coordinate for mixing layer
θ	= momentum thickness
λ	= wave length
λ_c	= wave length scale constant
ν	= kinematic viscosity
$\vec{r}$	= separation vector in an isotropic space
ρ	= fluid density
σ	= empirical inverse length scale for transverse velocity
σ_0	= shear layer spread parameter
τ	= time separation

Subscripts and Superscripts

A	= axial
c	= convection
i, j, k	= Cartesian tensor notation; i, j , or k equal to 1, 2, 3 corresponds to the axial, radial and transverse directions, respectively.
L	= convected frame
m	= measured
N	= normal

p	= pressure
p_k	= local peak value
TT	= turbulent-turbulent
TM	= turbulent-mean shear
u, v, w	= axial, radial, and transverse velocity
x, y, z	= axial, radial, and transverse directions
$\rightarrow$	= vector quantity

Overscores denote time averaged values

Primes denote rms values

1. INTRODUCTION

Turbulent flows in which velocity and pressure are stochastic variables are common in a wide variety of engineering applications. The role of fluctuating pressure in these flows is not fully understood; however, it is believed to play an important part in determining the structure and transport properties of the flows. Hinze (1959) and Tennekes and Lumley (1972) discuss the importance of the pressure momentum transfer mechanism in determining the structure of non-isotropic turbulence. Hinze (1959) points out the need for an understanding of pressure momentum transport for incorporation into turbulent mixing models. The experimental results of Crow and Champagne (1971) and Binder and Favre-Marient (1973) indicate large changes in jet mixing with upstream pressure perturbations. These studies suggest that pressure can be associated with jet stability and can be significant in determining the overall nature of the turbulent flow.

Two engineering application areas of current interest require a knowledge of the random pressure fluctuations associated with a turbulent field. The random pressure field near a surface can be a significant forcing term for structural vibrations. The models for nuclear reactor fuel pin vibration proposed by Paidousis (1966) and more recently refined by Kanazawa (1969) and Chen and Wambsganss (1972) use the surface pressure space-time correlation as a vibration forcing term.

The other area of considerable concern is aerodynamic noise generated by regions of intense turbulence. Lighthill (1952,1954)

described the remote pressure field (the far field sound) generated by localized regions of turbulence in terms of the turbulent velocity structure. Ribner (1964) extended this theory and described a jet's far field sound in terms of the hydrodynamic pressure within the mixing region turbulence. Ribner's theory suggests that an investigation of the fluctuating pressure in the mixing region of a jet would enhance understanding of turbulent sound generation and provide methods for calculating far field pressure distributions.

With both fundamental interests and direct applications in mind, the area defined for this study was the mixing region provided by a subsonic isothermal round jet. The turbulent mixing region is a fundamental model for a number of flow regimes, hence the results should have broad implications. The turbulent velocity in a mixing region has been extensively studied, so many of its characteristics are understood and experimental comparisons are available. The simplicity of this flow regime makes some theoretical predictions possible. Although limited by velocity errors in some flows, the pressure transducer operates ideally in the jet mixing region. Last, but of prime importance, both the fundamental understanding and data output of a round jet mixing region study are directly applicable to the problem of aerodynamic noise generation.

This report of the investigation proceeds from the theoretical dependence of the pressure field upon the velocity field, through an analysis and description of the pressure transducer, to the measurement and interpretation of the velocity and pressure fields.

As recognized and developed by Batchelor (1951) and others, fluctuating static pressure is not an independent variable in a turbulent flow. It is explicitly determined by the surrounding velocity field. This dependence in an incompressible Stokesian fluid is the basis for the theory review and development in Chapter 2. The theory is simplified to a mixing layer and a term dependent on the turbulent-mean shear interaction is isolated and developed. The results identify the features of the velocity field which most directly influence hydrodynamic pressure intensities. The results also form a hierarchy of predictions and scaling laws for pressure intensity.

Measurement of pressure was preceded by a lengthy development of a pressure transducer. In Chapter 3, a review of previous investigations identifies a number of instrument problems, the most serious of which is a direct response to the turbulent velocity field. The analysis and experimental verification presented shows the instrument used not only meets the general requirements of small size, good sensitivity, and flat frequency response, but it reduces the direct velocity field response to acceptably small levels. Chapter 4 describes the jet flow facility, the hot-wire anemometry, and the measurement system and techniques used to gather the large amount of data necessary for a comprehensive description of the mixing region pressure field.

The velocity field results, in Chapter 5, center around quantities identified by the theory as necessary to compute and scale hydrodynamic pressure intensity. These results include measurements and models for

mean velocity gradient, transverse velocity scales and intensities, and the transverse velocity three-dimensional space correlation.

The principal results of the study are presented in Chapter 6. The measured pressure intensities compare well with predictions based on the theory from Chapter 2 and the velocity field model from Chapter 5. Pressure intensity scaling laws are compared to experimental results and their range of applicability is established. The two-point pressure measurements show the structure of correlated volumes of pressure and their time evolution in the convected frame. A simple model for the four-dimensional pressure space-time correlation is proposed. The model incorporates and illustrates many of the important features of the correlation and should form a foundation for future applications to the problem of aerodynamic noise generation.

2. THEORETICAL ESTIMATES OF THE FLUCTUATING STATIC PRESSURE FIELD

The purpose of this chapter is to review the basic theory relating turbulent flow pressure and velocity and to extend and specify this theory to provide intensity estimates and scaling laws for turbulent mixing layers. In proceeding from the general governing equations to the specific mixing layer intensities, the necessary assumptions have been applied in steps, thus showing the generality of intermediate conclusions, and providing well-defined departure points for future studies of other aspects of the pressure field.

The review is selective and focuses only on issues directly related to free mixing layers. It relies heavily upon the early work of Batchelor (1951), Uberoi (1954), and Kraichnan (1956). The theory is specified and applied to a mixing layer using an empirical velocity space correlation model. The result is a prediction of, and scaling laws for, the turbulent-mean shear pressure intensity in a free mixing layer.

2.1 General Incompressible Formulation

The basis for understanding the pressure field in a Stokesian fluid is the conservation of linear momentum and mass expressed in the Navier-Stokes and continuity equations:

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j^2 \partial x_j} \quad (2-1)$$

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (2-2)$$

Incompressibility is assumed and Cartesian tensor notation with implied summation convention is used. Differentiation, with respect to X_i , of the momentum equation and use of continuity results in a Poisson-type equation for pressure

$$\frac{\partial^2 p}{\partial X_i^2} = -\rho \frac{\partial^2 u_i u_j}{\partial X_i \partial X_j} \quad (2-3)$$

The solution of Eq. 2-3 is:

$$p(\vec{x}, t) = -\frac{\rho}{4\pi} \int_{\vec{y}} \frac{d\vec{y}^3}{|\vec{y} - \vec{x}|} \frac{\partial^2 u_i u_j(\vec{y}, t)}{\partial y_i \partial y_j} \quad (2-4)$$

This result shows that the pressure follows the source term without time lag. This approximation is a result of the incompressibility assumption and the resulting Poisson equation. For stationary flows, Eq. 2-4 may be manipulated to produce the pressure space-time covariance.

$$\overline{p(\vec{x}, t) p(\vec{x}', t)} = \frac{\rho^2}{16\pi^2} \int_{\vec{y}} \frac{d\vec{y}^3}{|\vec{y} - \vec{x}|} \frac{\partial^2 u_i u_j(\vec{y}, t)}{\partial y_i \partial y_j} \int_{\vec{y}'} \frac{d\vec{y}'^3}{|\vec{y}' - \vec{x}'|} \frac{\partial^2 u_k u_l(\vec{y}', t)}{\partial y'_k \partial y'_l} \quad (2-5)$$

The overbars denote a statistical realization, or for stationary flows, a long time average. The primes denote second positions in space and time. For stationary flows, Eq. 2-5 is cast into the more common form:

$$\overline{p(\vec{x}, t) p(\vec{x}', t')} = \frac{\rho^2}{16\pi^2} \int_{\vec{y}} \frac{d\vec{y}^3}{|\vec{x} - \vec{y}|} \int_{\vec{y}'} \frac{d\vec{y}'^3}{|\vec{x}' - \vec{y}'|} \frac{\partial^2 u_i(\vec{y}, t) u_j(\vec{y}, t) u_k(\vec{y}', t') u_l(\vec{y}', t')}{\partial y_i \partial y_j \partial y'_k \partial y'_l} \quad (2-6)$$

Without loss of generality, the velocity may be divided into mean and fluctuating components $u_i = \bar{u}_i + u'_i$. Following some algebra, the

velocity tensoral source separates into the turbulent-turbulent and turbulent-mean shear components.

$$\overline{p(x,t)p(x',t')} = P_{TT}^2 + P_{TM}^2 \quad (2-7)$$

where

$$P_{TT}^2 = \frac{\rho^2}{16\pi^2} \int \frac{d\vec{y}^3}{|\vec{y}-\vec{x}|} \int \frac{d\vec{y}'^3}{|\vec{y}'-\vec{x}'|} \frac{\partial^2}{\partial y_i \partial y_j \partial y_k \partial y_l} \left[\overline{u_i u_j u_k u_l} - \overline{u_i u_j} \overline{u_k u_l} \right]$$

and

$$P_{TM}^2 = \frac{4\rho^2}{16\pi^2} \int \frac{d\vec{y}^3}{|\vec{y}-\vec{x}|} \int \frac{d\vec{y}'^3}{|\vec{y}'-\vec{x}'|} \frac{\partial \overline{u_i}}{\partial y_j} \frac{\partial \overline{u_k}}{\partial y_l} \frac{\partial^2}{\partial y_k \partial y_l'} \overline{u_j' u_l'}$$

Equation 2-7 is the starting point for analysis of pressure in several different flow fields. Batchelor (1951) and Uberoi (1954) have studied the turbulent-turbulent term extensively for isotropic turbulence. Kraichnan (1956) generalized these analyses and included scale anisotropy effects. These analyses, as well as those following below, assume local homogeneity of the turbulence. This assumption requires the spatial scale of variations, in turbulent intensity and scale, to be large compared to the turbulence scales. The additional assumption of joint normal velocity distributions allows reduction of the 4th order velocity correlation to second-order correlation functions. After the following transformation, homogeneity and stationarity imply the correlations are functions only of separations in space $\vec{r}$, and separations in time τ .

$$\begin{aligned}
\vec{y} &= \vec{y} \\
\vec{r} &= \vec{y}' - \vec{y} \\
\tau &= t - t'
\end{aligned}
\tag{2-8}$$

Joint normality allows

$$\overline{u_i u_j u_k u_l} = \overline{u_i u_j} \overline{u_k u_l} + \overline{u_i u_k} \overline{u_j u_l} + \overline{u_i u_l} \overline{u_j u_k}
\tag{2-9}$$

With these assumptions, Kraichnan (1956) shows the P_{rr}^2 portion of Eq. 2-7 may be integrated over the position coordinate $\vec{y}$ resulting in

$$P_{rr}^2(\vec{x}, \tau) = -\frac{\rho^2}{8\pi} \int \frac{d\vec{r}}{r^3} \frac{\vec{r}}{r} \frac{1}{2^4} \frac{(\overline{u_i u_l} \overline{u_j u_k} + \overline{u_i u_k} \overline{u_j u_l})}{2^2 r_i^2 r_j^2 r_k^2 r_l^2}
\tag{2-10}$$

where the notation change $\vec{x}' - \vec{x} \rightarrow \vec{x}$ is used for compactness.

2.2 Pressure from Turbulent-Turbulent Interaction

The turbulent-turbulent contribution will now be considered for the case of $\mathcal{U} = 0$. Batchelor (1951) assumed isotropic turbulence so the two-point correlation could be written in terms of the single-defining scalar $f(r)$.

viz.

$$\overline{u_i u_j}(r) = \overline{u^2} \left\{ -\frac{1}{2r} \frac{\partial f(r)}{\partial r} r_i r_j + \left[f(r) + \frac{r}{2} \frac{\partial f(r)}{\partial r} \right] \delta_{ij} \right\}
\tag{2-11}$$

where $f(r) = \overline{u_i u_i}(r_i)$ (no summation)

Inserting Eq. 2-11 into Eq. 2-10 for the turbulent-turbulent source and carrying out the integrations results in

$$P_{rr}^2(x) = 2\rho^2(\overline{u^2})^2 \int_x^\infty d\mathcal{R} \left(\mathcal{R} - \frac{x^2}{\mathcal{R}} \right) \left(\frac{\partial f(\mathcal{R})}{\partial \mathcal{R}} \right)^2
\tag{2-12}$$

Several important results follow from Eq. 2-12 with experimentally determined or assumed forms for $f(s)$. The pressure-space correlation decreases more rapidly than the f function or alternately the pressure space scale is smaller than the velocity space scale. Uberoi (1954) has calculated pressure intensity with experimentally determined velocity correlations for a range of turbulent Reynolds numbers. His results show

$$(p_{\tau\tau}^2(0))^{1/2} \simeq 0.7 \rho \bar{U}^2$$

For later comparison, if $f = \bar{U}^2 e^{-\sigma s}$ then

$$(p_{\tau\tau}^2(0))^{1/2} = \sqrt{2} \left(\frac{1}{2} \rho \bar{U}^2 \right) \quad (2-12A)$$

Kraichnan (1956) later estimated the $p_{\tau\tau}^2$ contribution for non-isotropic turbulence using the scale anisotropy concept to be discussed later. The results of this analysis for an assumed $f = \bar{U}^2 e^{-\sigma s}$ are:

$$(p_{\tau\tau}^2)^{1/2} = \sqrt{2} \left(\frac{1}{2} \rho \bar{U}^2 \right) \sqrt{K(\alpha)} \quad (2-12B)$$

In Eq. 2-12B, α is the ratio of turbulence scales in the flow and transverse directions. $K(\alpha)$ is a decreasing function of α to be discussed later.

2.3 Pressure from Turbulent-Mean Shear Interaction

The $p_{\tau m}^2$ term will now be considered. Following Kraichnan, we assume the mean flow is unidirectional in the 1 direction and that mean

variations exist only in the 2 direction. The P_{rm}^2 term in Eq. 2-7, evaluated at $x = x'$, and $t = t'$ for the point mean square pressure, then reduces to:

$$P_{rm}^2(0) = \frac{4\rho^2}{16\pi^2} \int \frac{d\vec{y}^3}{|\vec{y}|} \int \frac{d\vec{y}'^3}{|\vec{y}'|} \frac{\partial \bar{U}_1(y_2)}{\partial y_2} \frac{\partial \bar{U}_1(y'_2)}{\partial y'_2} \frac{\partial^2 \overline{U_2 U_2}(\vec{y}, \vec{y}')}{\partial y_1 \partial y'_1} \quad (2-13)$$

It is important to note that in this relatively general form the P_{rm}^2 depends only on the mean velocity gradient and the transverse velocity correlation. The transformation in Eq. 2-8 yields:

$$P_{rm}^2 = \frac{4\rho^2}{16\pi^2} \int \frac{d\vec{y}^3}{|\vec{y}|} \int \frac{d\vec{s}^3}{|\vec{s}-\vec{y}|} \frac{\partial \bar{U}_1(y_2)}{\partial y_2} \frac{\partial \bar{U}_1(y_2+s_2)}{\partial s_2} \left(-\frac{\partial^2 \overline{U_2 U_2}(y, s)}{\partial s_1^2} + \frac{\partial^2 \overline{U_2 U_2}(y, s)}{\partial s_1 \partial y_1} \right) \quad (2-14)$$

Homogeneous turbulence with a constant mean velocity gradient is assumed. This allows the following approximations:

$$\frac{\partial}{\partial y_1} \overline{U_2 U_2}(y, s) \sim 0 \quad (2-15A)$$

$$\frac{\partial^2}{\partial s_1^2} \overline{U_2 U_2}(y, s) \sim \frac{\partial^2}{\partial s_1^2} \overline{U_2 U_2}(s) \quad (2-15B)$$

$$\frac{\partial \bar{U}_1}{\partial s_2}(y_2+s_2) \sim \frac{\partial \bar{U}_1}{\partial y_2}(y_2) \quad (2-15C)$$

and

$$\int \frac{d\vec{y}^3}{|\vec{y}| |\vec{s}-\vec{y}|} \left(\frac{\partial \bar{U}_1(y_2)}{\partial y_2} \right)^2 \frac{\partial^2 \overline{U_2 U_2}(y, s)}{\partial s_1^2} \sim \left(\frac{\partial \bar{U}_1(y_2)}{\partial y_2} \right)^2 \frac{\partial^2 \overline{U_2 U_2}(s)}{\partial s_1^2} \int \frac{d\vec{y}^3}{|\vec{y}| |\vec{s}-\vec{y}|} \quad (2-15D)$$

Equation 2-15A is nearly exact in self similar flows in which changes in the flow direction are from a slowly increasing turbulent scale. Equations 2-15B, C, D are plausible near a mixing centerline where scales, intensities, and gradients once again experience slow flow direction variation due to scale increase and moderate variation with transverse position. The result of this hypothesis is:

$$P_{TM}^2 = -\frac{\rho}{4\pi^2} \left(\frac{\partial \overline{U_1}}{\partial y_2} \right)^2 \int_{\vec{S}} d\vec{S}^3 \frac{\partial^2 \overline{U_2 U_2}}{\partial S_1^2} \int \frac{d\vec{y}^3}{|\vec{y}| |\vec{S} - \vec{y}|} \quad (2-16)$$

Kraichnan (1956) showed the position could be integrated yielding:

$$P_{TM}^2 = \frac{\rho^2}{2\pi} \left(\frac{\partial \overline{U_1}}{\partial y_2} \right)^2 \int_{\vec{S}} d\vec{S}^3 \frac{\partial^2 \overline{U_2 U_2}(\vec{S})}{\partial S_1^2} \vec{S} \quad (2-17)$$

A double partial integration yields:

$$P_{TM}^2 = \rho^2 \left(\frac{\partial \overline{U_1}}{\partial y_2} \right)^2 \int_{\vec{S}} d\vec{S}^3 \left(1 - \frac{S_1^2}{S^2} \right) \frac{1}{S} \overline{U_2 U_2}(S) \quad (2-18)$$

Given a velocity correlation, Eq. 2-18 may be used to calculate the P_{TM}^2 term. Three models for turbulent velocity correlation have been examined. They are Kraichnan's (1956) elongated isotropic model, an empirical model based on isotropic functional forms, and Goldstein's (1973) axisymmetric turbulence forms. The axisymmetric model may form a basis of a more general theory; hence the implications are explored

in Appendix I. At this time, however, a depth of experimental information necessary for axisymmetric implementation is not available.

Thus, Kraichnan's work, experimentally verifiable parts of which form the empirical model, will be discussed and extensively used.

2.4 Kraichnan Scale Anisotropy Turbulence Model

In isotropic turbulence, the transverse velocity covariance may be expressed in terms of a single-defining scalar function of separations as expressed in Eq. 2-11. The scale anisotropy model considers the same form in a spatial frame elongated in the flow direction. The elongation scale transformation is

$$\begin{aligned}\mathcal{J}_1 &= \alpha \mathcal{J}_1 \\ \mathcal{J}_2 &= \mathcal{J}_2 \\ \mathcal{J}_3 &= \mathcal{J}_3\end{aligned}\tag{2-19}$$

The covariances transform by:

$$\begin{aligned}R_{11}(\mathcal{J}) &= \alpha^2 R_{11}(\mathcal{J}) \\ R_{22}(\mathcal{J}) &= R_{22}(\mathcal{J}) \\ R_{33}(\mathcal{J}) &= R_{33}(\mathcal{J})\end{aligned}\tag{2-20}$$

And the kinetic energies are related by:

$$KE(\mathcal{J}) = KE(\mathcal{J}) \frac{\alpha^2 + 2}{3}\tag{2-21}$$

The length relation below is also important.

$$\mathcal{J}^2 = \mathcal{J}^2 - \frac{\alpha^2}{\alpha^2} \mathcal{J}_1^2\tag{2-22}$$

Note $\mathcal{J}$ meters physically realizable space observed in an experiment.

In this space, compared to the isotropic space metered by ξ , scales and velocities in the flow direction are increased by a factor of α and kinetic energy is modified by $\frac{\alpha^2+2}{3}$. Kraichnan (1956) remarks that although kinematically possible, the elongated isotropic form is not theoretically complete because it would be unstable under the Navier-Stokes inertial forces. This deficiency and an alternate empirical model will be discussed in the next section. A functional form for the defining scalar is now chosen as

$$f(\xi) = e^{-\sigma \xi} \quad (2-23)$$

This form, which is different than that assumed by Kraichnan, has physical significance and is observed experimentally except for $\sigma \xi \ll 1$. According to Eq. 2-11, the transverse isotropic correlation is

$$\overline{u_2 u_2}(\xi) = e^{-\sigma \xi} \left(1 - \frac{1}{2} \sigma \xi + \frac{1}{2} \frac{\sigma^2 \xi^2}{\xi} \right) \overline{U^2} \quad (2-24)$$

where $\overline{U^2}$ is an isotropic mean square velocity. The scale transformation, applied to Eq. 2-24, yields

$$\overline{u_2 u_2}(\xi) = \overline{U^2} e^{-\sigma(\xi^2 - \frac{\alpha^2}{\alpha^2} \xi^2)^{1/2}} \left[1 - \frac{1}{2} \sigma(\xi^2 - \frac{\alpha^2}{\alpha^2} \xi^2)^{1/2} + \frac{1}{2} \frac{\sigma^2 \xi^2}{(\xi^2 - \frac{\alpha^2}{\alpha^2} \xi^2)^{1/2}} \right] \quad (2-25)$$

The velocity correlation model in Eq. 2-25 is substituted into Eq. 2-18 to calculate P_{TM}^2 , and, following Kraichnan, the result is normalized by $\frac{3}{2+\alpha^2}$ to compare with the isotropic case.

$$P_{TM}^2 = \frac{\rho^2}{2\pi} \left(\frac{2U_1}{2Y_1} \right)^2 \frac{\bar{U}^2 3}{2+\alpha^2} \int_{\vec{g}} d\vec{g}^3 \left\{ e^{-\sigma \left(g^2 - \left(\frac{\alpha^2}{\alpha^2} \right) g_1^2 \right)^{1/2}} \left[1 - \frac{1}{2} \sigma \left(g^2 - \frac{\alpha^2}{\alpha^2} g_1^2 \right)^{1/2} + \right. \right. \\ \left. \left. + \frac{1/2 \sigma g_2^2}{g^2 - \left(\frac{\alpha^2}{\alpha^2} \right) g_1^2} \right] \left[1 - \frac{g_1^2}{g^2} \right] \frac{1}{g} \right\} \quad (2-26)$$

The integration in Eq. 2-26 may be performed most easily by using the spherical transformation

$$\begin{aligned} g_1 &= r \cos \varphi \\ g_2 &= r \sin \varphi \cos \theta \\ g_3 &= r \sin \varphi \sin \theta \end{aligned} \quad (2-27)$$

so that

$$P_{TM}^2 = \frac{\rho^2}{2\pi} \frac{2U_1}{2Y_2} \frac{\bar{U}^2 3\alpha}{2+\alpha^2} \int_0^\infty dr R^2 \int_0^\pi d\varphi \sin \varphi \int_0^{2\pi} d\theta \left\{ \mathcal{J} \right\} \\ \mathcal{J} = \left\{ e^{-\sigma r} \left[1 - \frac{1}{2} \sigma r + \frac{1/2 \sigma R^2 \sin^2 \varphi \cos^2 \theta}{R} \right] \left[1 - \frac{\alpha^2 R^2 \cos^2 \varphi}{R^2 + (\alpha^2) R^2 \cos^2 \varphi} \right] \left[\frac{1}{R^2 + (\alpha^2) R^2 \cos^2 \varphi} \right]^{1/2} \right\} \quad (2-28)$$

Integrations over θ , R , and φ yield:

$$P_{TM}^2 = \left[8/15 \left(\frac{2U_1}{2Y_2} \right)^2 \rho^2 U_2^2 \frac{1}{\sigma^2} \right] [J(\alpha)] \quad (2-29)$$

The first bracketed term is the isotropic result. The second, $J(\alpha)$, is an anisotropy factor and is given by

$$J(\alpha) = \begin{cases} \frac{45}{16(2+\alpha^2)} \left[\alpha^2 + \psi + 4 \left(\frac{1-\psi}{\gamma^2} \right) + (1-2\gamma^2) \frac{2+\alpha^2-3\psi}{\gamma^4} \right] & ; \alpha \neq 1.0 \\ 1.0 & ; \alpha = 1.0 \end{cases} \quad (2-30)$$

where

$$\gamma^2 = |1 - \alpha^2|$$

$$\psi = \begin{cases} \alpha \cosh^{-1}(\alpha) / (\alpha^2 - 1)^{1/2} & ; \alpha > 1.0 \\ \alpha \cos^{-1}(\alpha) / (1 - \alpha^2)^{1/2} & ; \alpha < 1.0 \end{cases}$$

The $J(\alpha)$ factor modifies the isotropic intensity to account for scale anisotropies. It is the same factor found by Kraichnan for the defining scalar $f = e^{-\sigma^2 \gamma^2}$. Further, as demonstrated by Kraichnan, for α on the order of a small integer it is about the same size as the ρ_{rr}^2 anisotropy factor $K(\alpha)$.

2.5 An Empirical Velocity Space Correlation Model

As previously discussed, there are fundamental objections to an isotropic-type model for shear flow correlation representation. Experimental evidence in Chapter 5 substantiates these objections. As one example of practical importance, it was found that $\left(\frac{\overline{u_1^2}}{\overline{u_2^2}} \right)^{1/2}$, $\frac{L_1}{L_2}$, and $\frac{L_1}{L_3}$ were not equal. Thus, a choice of α was ambiguous and the model must be restricted and its interpretation clarified prior to use.

The empirical model requirements for predicting ρ_{rr}^2 follow directly from Eq. 2-17. The correlation of the transverse velocity component must be known for separations in all directions. No knowledge of other velocity components is required. Equation 2-18 is taken here as an empirical model for $\overline{u_2 u_2}(\gamma)$, subject to experimental verification and parameter determination. The mean square fluctuating velocity is precisely the measured transverse velocity. The

parameters' notation is changed to unambiguously identify them as empirical model parameters. The result is

$$\overline{u_2 u_2}(\delta) = \overline{u_2^2} e^{-\sigma(\delta^2 - \frac{d_1^2}{\sigma^2} \delta_1^2)^{1/2}} \left\{ 1 - \frac{1}{2} \sigma(\delta^2 - \frac{d_1^2}{\sigma^2} \delta_1^2)^{1/2} + \frac{1/2 \sigma \delta_2^2}{(\delta^2 - \frac{d_1^2}{\sigma^2} \delta_1^2)^{1/2}} \right\} \quad (2-31)$$

The precise interpretation of the parameters is clear from the following special cases.

$$\overline{u_2 u_2}(\delta_2) = \overline{u_2^2} e^{-\sigma \delta_2} \quad (2-32)$$

$$\overline{u_2 u_2}(\delta_1) = \overline{u_2^2} e^{-\frac{\sigma \delta_1}{\sigma^2} (1 - \frac{1}{2} \frac{\sigma \delta_1}{\sigma^2})} \quad (2-33)$$

This model is inserted into Eq. 2-17 and integrated as before, except the kinetic energy scaling factor is not required. The result is

$$\rho_{TM}^2 = 8/15 \left(\frac{2\overline{u_1}}{2\overline{u_2}} \right)^2 \rho^2 \overline{u_2^2} \frac{1}{\sigma^2} J(d_1) \frac{2+d_1^2}{3} \quad (2-34)$$

In summary, the empirical model utilizes the same forms proposed by Kraichnan (1956). Its validity to be demonstrated in Ch. 5, rests solely upon its ability to represent experimental space correlations of the transverse velocity correlation. Because of this restriction the parameters are unambiguously identified as fit parameters for Eq. 2-31.

2.6 Similarity and Scaling Laws for Pressure Intensity

Scaling or normalizing engineering data is common. The objective is to express non-dimensional results in a widely applicable invariant form. Generally, there is a trade-off between generality, precision, and simplicity.

At the outset we restrict attention to systems for which our previous assumptions are applicable. Scaling for the turbulent-turbulent contribution then follows from Eq. 2-12. The turbulent-mean shear contribution scaling follows from Eq. 2-17. An essential difference between these contributions is observed when the integrals in the two equations are non-dimensionalized.

$$P_{TT}^2 \propto (\rho \bar{U}^2)^2 \quad (2-35)$$

$$P_{TM}^2 \propto \left(\frac{\rho \bar{U}}{2 \gamma_2} \right)^2 \frac{1}{\sigma^2} \bar{U}_2^2 \quad (2-36)$$

In Eq. 2-36, $\frac{1}{\sigma}$ is a characteristic length. These equations show the turbulent-turbulent contribution depends only on turbulent intensity, while the turbulent-mean shear depends on intensity, mean gradient, and length scale.

To provide more scaling definition, attention is now focused upon flows in which the mean shear contribution is assumed to dominate. For these flows, scaling follows from Eq. 2-34, or Eq. A-13, if axisymmetric details are available and desirable. A hierarchy of scalings may be derived from Eq. 2-34. Among the possibilities are the case for flows

in which the structure is invariant and the case for a self-preserving mixing layer. If the structure is invariant, then $J(\delta) \frac{2+\delta^2}{3} = \text{constant}$. Consequently,

$$\frac{(p_{TM}^2)^{1/2}}{\frac{\partial u_1}{\partial y_2} \rho (\overline{u_2^2})^{1/2} \frac{1}{\sigma}} = \text{constant} \quad (2-37)$$

The case for a modeled mixing layer follows. We assume a two-stream mixing layer in which the free stream velocities are U_a and U_b . The velocity ratio is defined as $k = \frac{U_b}{U_a}$. The mean velocity gradient is assumed to vary with axial position and free stream velocity difference according to

$$\frac{\partial \bar{u}_1}{\partial y_2} = \frac{\sigma_0}{\sqrt{\pi}} \frac{U_a - U_b}{x - x_0} \quad (2-38)$$

In Eq. 2-38 σ is a constant. The assumed scale variation is given by

$$\frac{1}{\sigma} = L_0 \frac{1-k}{1+k} (x - x_0) \quad (2-39)$$

where L_0 is a scale constant. The turbulent structure is assumed constant so that δ and $J(\delta)$ are constants. With these assumptions, the scaling law follows from Eq. 2-37.

$$\frac{(p_{TM}^2)^{1/2}}{\rho U_a (\overline{u_2^2})^{1/2} \left(\frac{1-k}{1+k} \right)^2} = \text{constant} \quad (2-40)$$

If the ratio $\frac{U_2^2}{(U_a - U_b)^2}$ is constant, then

$$\frac{(p_{TM}^2)^{1/2}}{\rho U_a^2 \frac{(1-k)^3}{(1+k)^2}} = \text{constant} \quad (2-41)$$

For a simple jet mixing layer $k = 0$. Thus,

$$\frac{(p_{TM}^2)^{1/2}}{\rho U_a^2} = \text{constant} \quad (2-42)$$

In summary, the incompressible Navier-Stokes and continuity equations show the pressure is completely determined by the surrounding velocity field. The fluctuating pressure was divided into a component from turbulent-turbulent interaction and a component from turbulent-mean shear interaction. These components add in quadrature.

The turbulent-turbulent component is a function of velocity intensity. Assuming locally homogeneous turbulence, the turbulent-turbulent pressure space scales are smaller than the turbulent velocity space scales. In contrast, the turbulent-mean shear intensity is a function of transverse turbulent velocity intensity, velocity space scale, and the mean velocity gradient. A series of simplifying assumptions allowed explicit determination of the turbulent-mean shear pressure in terms of an empirical velocity field model. This model is practical and precise as will be shown in Chapter 5. As will be shown in Chapter 6, this pressure intensity theory compares well with

experimental results, particularly near the mixing center where the simplifying assumptions are most correct.

3. FLUCTUATING STATIC PRESSURE MEASUREMENT

The measurement of fluctuating static pressure has long been of great interest. Efforts, however, have been hampered by the lack of a suitable sensor. The requirements for such a sensor are rather severe. It must measure pressure in, and present minimal disturbance to, a flow volume small compared to the turbulent scale. The amplitude frequency response should be flat in the range of interest (the audio range for our purposes). Additionally, for multi-sensor measurements, phase lags should be small. Finally, the direct response for the fluctuating velocity field should be negligibly small. A miniature pressure transducer (PT) has been developed which appears to meet these requirements.

3.1 Historical Survey

The previous list of requirements for a pressure-measuring device follows from the experience of several earlier investigations. Spencer (1970) gives an excellent review of early attempts and difficulties in measuring pressure. Generally, early instruments used piezo electric or capacitance microphone type pressure pick-ups mounted in a static pressure tube. As the size of this type of instrument is reduced to less than turbulent scale size, the signals generally decrease and further resonances appear in the audio range. This size restraint was a problem for Strasburg and Cooper (1965), Kobashi (1957), and Sami (1967). Even with recent sensitivity improvements to small capacitance microphones, resonances must be damped as reported by Arndt and Nilsen (1971).

The other general problem is caused by probe-velocity field interaction. To cope with this problem, Rouse (1954) placed pressure sensing holes at 47° from stagnation on a spherical tip. At this position, ideal flow analysis shows the pressure coefficient to be small. His probe, however, was sensitive to alignment and was large (1/4 in. diameter). Such a tip was tested during the early stages of our investigation. This concept was abandoned because the smallest holes acceptable (.013 in. diameter) were large compared with the miniature tip (.065 in. diameter), and consequently, caused a serious flow disturbance. Additionally, the machining problems of this type of tip were severe and reproducibility questionable.

Most other investigators have used a classic static pressure tube design to balance nose and stem acceleration and produce a zero pressure coefficient for axial velocity. This geometry, however, is still sensitive to transverse velocity fluctuations. The sensitivity of a static pressure tube to transverse velocity has been quantified by several investigators. Goldstein's (1936) estimate assumed the tube was much larger than the turbulent eddies. The work of Toomre (1960) provides a cross-flow pressure coefficient for both the large and small turbulent scale limit. Siddon (1969) empirically formulated the large scale cross flow pressure coefficient for various Reynolds numbers. Later he developed a probe which measured both pressure and cross velocity components and corrected for cross velocity errors. The correction scheme was complex and in shear flow the corrections were small, seemingly only slightly larger than uncertainties in the correction.

Our approach has been to adapt the bleed-type transducer, developed by Spencer (1970), for measurements in large scale, high speed, free shear layers. In these flows, a large scale non-viscous flow analysis and experimental verification clearly provides the cross-flow pressure coefficient and estimates of velocity-induced error. The error in pressure intensity is acceptably small.

3.2 Pressure Transducer Description

The pressure transducer is shown in Fig. 3.2-1. The structural and design details are given by Spencer and Jones (1971). The transducer operates from bleeding air in laminar flow through a capillary tube from a constant pressure reservoir. Pressure fluctuations at the capillary downstream end set up fluctuating velocities in the bleed fluid, which are measured by a hot film sensor mounted across the exit of the capillary tube. The pressure transducer is fitted with a static pressure tip to minimize response to velocity fluctuations. The bleed fluid flows through the tip and exits via six circumferential bleed holes.

The pressure transducer meets the requirements that its size must be much smaller than the turbulent space scales of interest. For example, Laurence (1956) presents measurements that within the mixing region of a round jet the transverse velocity space scale $L_r \approx 0.04x$. Here x is the axial distance from the lip of the jet. At $x = 10$ in., $L_r = 0.4$ in. In comparison, the diameter of the transducer is 0.06 in. Evidence in Chapter 6 shows pressure scales even larger than velocity scales. The well contoured, highly polished tip presents a minimum flow

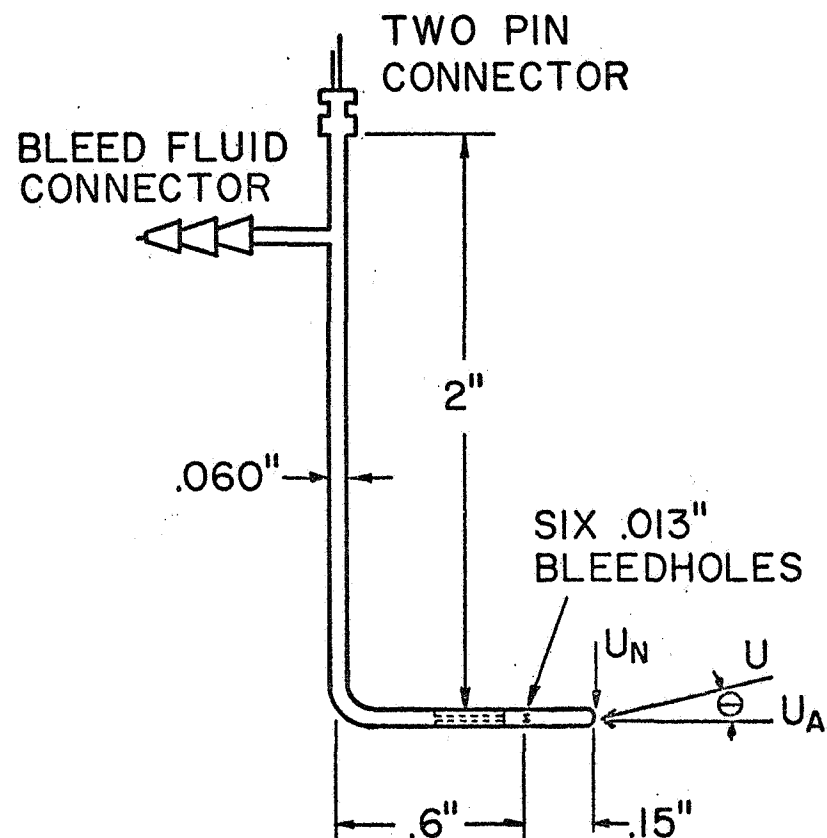
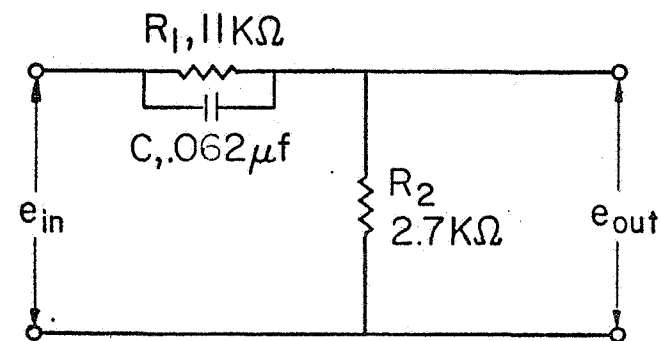


Fig. 3.2-1 Pressure Transducer.



$$\left(\frac{e_{out}}{e_{in}}\right)^2 = \frac{1 + (\omega/\omega_c)^2}{\alpha^2 + (\omega/\omega_c)^2}$$

$$\alpha = \frac{R_1 + R_2}{R_2}$$

$$\omega_c = 1 / R_1 C$$

Fig. 3.2-2 Frequency Response Correcting Circuit.

disturbance. The low bleed rate results in a very low volumetric injection rate of bleed fluid into the free stream (0.001 cubic inches/sec. per bleed hole). At very low free stream velocities (less than 50 fps) and high bleed rates (0.003 cubic inches/sec.), the effect of the bleed injection is seen as thickening of the probe boundary layer which results in a sensitivity to axial velocity fluctuations. However, at free stream velocities of interest (above 75 fps) and with lower bleed rates, this effect is not discernible.

The frequency response of the pressure transducer is fixed by the size of the capillary tube and the hot film sensor. The inner diameter of the capillary was chosen using Uchida's (1956) results for the response of a viscous fluid in tube flow to a time varying pressure gradient. The diameter is nominally .007 in. The cavity volume and the diameter of bleed holes in the diffuser tip were sized to avoid acoustical resonance or filtering within the audio range. To ensure matched transducers, a detailed test was developed to ensure proper sizing and flow resistance of the capillary tube. With this test, the manufacturer was able to construct reproducible and matched transducers.

The frequency response test, to be discussed later, showed the PT response dropping at about 4 dB per octave above 400 Hz. Two matched resistance capacitance (RC) circuits as shown in Fig. 3.2-2 were added to flatten the response. The circuits were designed by selecting α to produce a rise of about 4 dB per octave and selecting ω_c to match the PT drop-off frequency. The design upper frequency limit of the 4 dB per octave is asymptotically approached so that above 10 kHz little or no correction is applied.

3.3 Pressure Transducer Velocity-Induced Error Theory

The largest problem in measuring fluctuating static pressure arises from the interaction of the fluctuating flow field with the surface of the instrument. Siddon's (1969) investigation and Fuch's (1972) analysis indicates that this error for our pressure transducer can be modeled as a quasistatic, ideal flow, velocity-to-pressure conversion. The analytical and experimental results which follow show this is an applicable model, and that errors from this mechanism are small in a high-speed mixing layer.

The essential assumptions of the model are:

- a. The pressure transducer measures the pressures developed on its surface due to velocity-to-pressure conversion and static pressure in a linear combination.
- b. The size of the surface area in question is small compared to the scale of velocity spatial variations.
- c. The surface pressure due to velocity develops in a very short time and flow separation does not occur.

With these assumptions, the surface pressure distribution follows easily from a steady-state ideal flow analysis. Since the describing equation is linear, the effects of axial velocities and normal velocities can be separated and analyzed individually.

The functional relationship between axial velocity and pressure at the holes of a static pressure tube was developed by Prandtl. Dean (1953) shows these results can be expressed as

$$P - P_{\infty} = A \frac{1}{2} \rho U A^2 \quad (3-1)$$

where P is the pressure at the sensing holes, U is the free stream axial velocity, and A is the pressure coefficient fixed by the accelerating influence of the hemispherical tip and the stagnating influence of the stem. Using figures in Dean (1953) that quantify the stem and tip effects, the pressure-sensing holes were located to produce $A \approx 0$.

The pressure distribution caused by the cross component of velocity was approximated as the aximuthal pressure distribution on a long cylinder in ideal cross flow. The solution for the pressure distribution is

$$P - P_{\infty} = \frac{1}{2} \rho V_N^2 (1 - 4 \sin^2 \phi) \quad (3-2)$$

where V_N is the velocity component normal to the surface of the pressure transducer and ϕ is the polar angle between V_N and the surface point in question. The effective pressure sensed is the average pressure of the six holes and can be shown to be

$$P - P_{\infty} = \frac{1}{2} B \rho V_N^2 ; B = -1.0 \quad (3-3)$$

It should be pointed out that for three or more holes the averaged result does not depend upon the relative orientation of ϕ and the holes. Combining Eq. 3-2 and 3-3 yields an expression for the pressure transducer response to the velocity field

$$E(t) = S_p (A \frac{1}{2} \rho U_A^2(t) + B \frac{1}{2} \rho V_N^2(t)) \quad (3-4)$$

$E(t)$ is the total voltage developed by the axial and normal fluctuating

velocities. S is the sensitivity of the probe to pressure and is a negative number.
 p

Equation 3-4 is developed to express the velocity-induced error in terms of the constants A and B and the velocity field structure. Substituting mean and fluctuating components into Eq. 3-4 and subtracting the mean component yields the fluctuating error

$$E_e(t) = E(t) - \overline{E(t)}$$

or

$$E_e(t) = \frac{S p A \rho}{2} (2 U_A(t) \overline{U_A} + U_A^2(t) - \overline{U_A^2(t)}) + \frac{S p B \rho}{2} (2 V_N(t) \overline{V_N} + V_N^2(t) - \overline{V_N^2(t)}) \quad (3-5)$$

The long time average, denoted by an overbar, of the mean square of Eq. 3-5 is now obtained and the following turbulent structural constants are substituted:

$$F_A = \frac{\overline{U_A^3}}{(\overline{U_A^2})^2} \quad F_N = \frac{\overline{V_N^3}}{(\overline{V_N^2})^2} \quad S = \frac{\overline{U_A^3}}{(\overline{U_A^2})^{3/2}}$$

$$I = \frac{(\overline{U_A^2})^{1/2}}{\overline{U_A}} \quad K_N = \left(\frac{\overline{V_N^2}}{\overline{U_A^2}} \right)^{1/2} \quad R_{V_N^2 U_A^2} = \frac{\overline{V_N^2 U_A^2}}{\overline{V_N^2} \overline{U_A^2}} \quad (3-6)$$

$$f = \frac{(\overline{D^2})^{1/2}}{1/2 \rho \overline{U_A^2}} = \frac{\rho'}{1/2 \rho \overline{U_A^2}} \quad R_{V_N^2 U} = \frac{\overline{V_N^2 U_A}}{K (\overline{U_A^2})^{3/2}}$$

The result is:

$$\overline{E_e^2} = S_p^2 \left\{ \begin{aligned} &A^2 \left(\frac{p}{2} \right)^2 (\overline{U_A^2})^2 \left[\frac{4}{I^2} + \frac{4S}{I} + F_A - 1 \right] + \\ &AB \left(\frac{p}{2} \right)^2 (\overline{U_A^2})^2 K_N^2 \left[\frac{R_{VN^2} U_A}{I} + R_{VN^2} U_A^2 - 1 \right] + \\ &B^2 \left(\frac{p}{2} \right)^2 (\overline{U_A^2})^2 \left[(F_N - 1) K_N^2 \right] \end{aligned} \right\} \quad (3-7)$$

where it is assumed that $\overline{V_N} = 0$, a condition realized in practice by aligning the probe's axis with the mean flow.

Assuming weak statistical connection between the velocity error and true static pressure response E_p , their ratio can be found.

$$\frac{\overline{E_e^2}}{\overline{E_p^2}} = \frac{1}{f^2} \left\{ \begin{aligned} &A^2 \left(\frac{4}{I^2} + \frac{4S}{I} + F_A - 1 \right) + \\ &AB K_N^2 \left(\frac{R_{VN^2} U_A}{I} + R_{VN^2} U_A^2 - 1 \right) + \\ &B^2 K_N^2 (F_N - 1) \end{aligned} \right\} \quad (3-8)$$

Equation 3-8 is evaluated using experimental data at several positions across the mixing layer. The normal velocity structural constants must be evaluated in terms of measured cartesian velocities. Thus using the fact that $V_n^2 = V^2 + W^2$ yields:

$$R_{V_N^2 U_A} = \frac{V^2 U_A}{V^2 U_A' (1 + \frac{\omega^2}{V^2})} + \frac{\omega^2 U_A}{\omega^2 U_A' (1 + \frac{V^2}{\omega^2})}$$

$$R_{V_N^2 U_A} = \frac{\overline{V^2 U^2}}{\overline{V^2} \overline{U^2} (1 + \frac{\omega^2}{V^2})} + \frac{\overline{\omega^2 U^2}}{\overline{\omega^2} \overline{U^2} (1 + \frac{V^2}{\omega^2})} \quad (3-9)$$

$$F_N = \frac{\overline{V^4}}{(\overline{V^2})^2 (1 + \frac{\omega^2}{V^2})} + \frac{\overline{V^2 \omega^2}}{\overline{V^2} \overline{\omega^2} (\frac{V^2}{\omega^2} + 2 + \frac{\omega^2}{V^2})} + \frac{\overline{\omega^4}}{(\overline{\omega^2})^2 (1 + \frac{V^2}{\omega^2})}$$

Equations 3-8 and 3-9 predict the error due to the fluctuating velocity field in terms of the probe pressure coefficients A and B and measured structural parameters of the turbulent velocity. An evaluation of this error is made in the next section.

3.4 Experimental Verification

As with any new instrument, an extensive series of tests were necessary to pick the operating conditions and establish the response of the pressure transducer. Two areas, the verification of the error theory and frequency response testing, were extensive and will be described separately.

3.4.1 General Characteristics

The transducer was operated with a standard constant temperature anemometer system as shown in Fig. 3.4-2. It was found to behave like

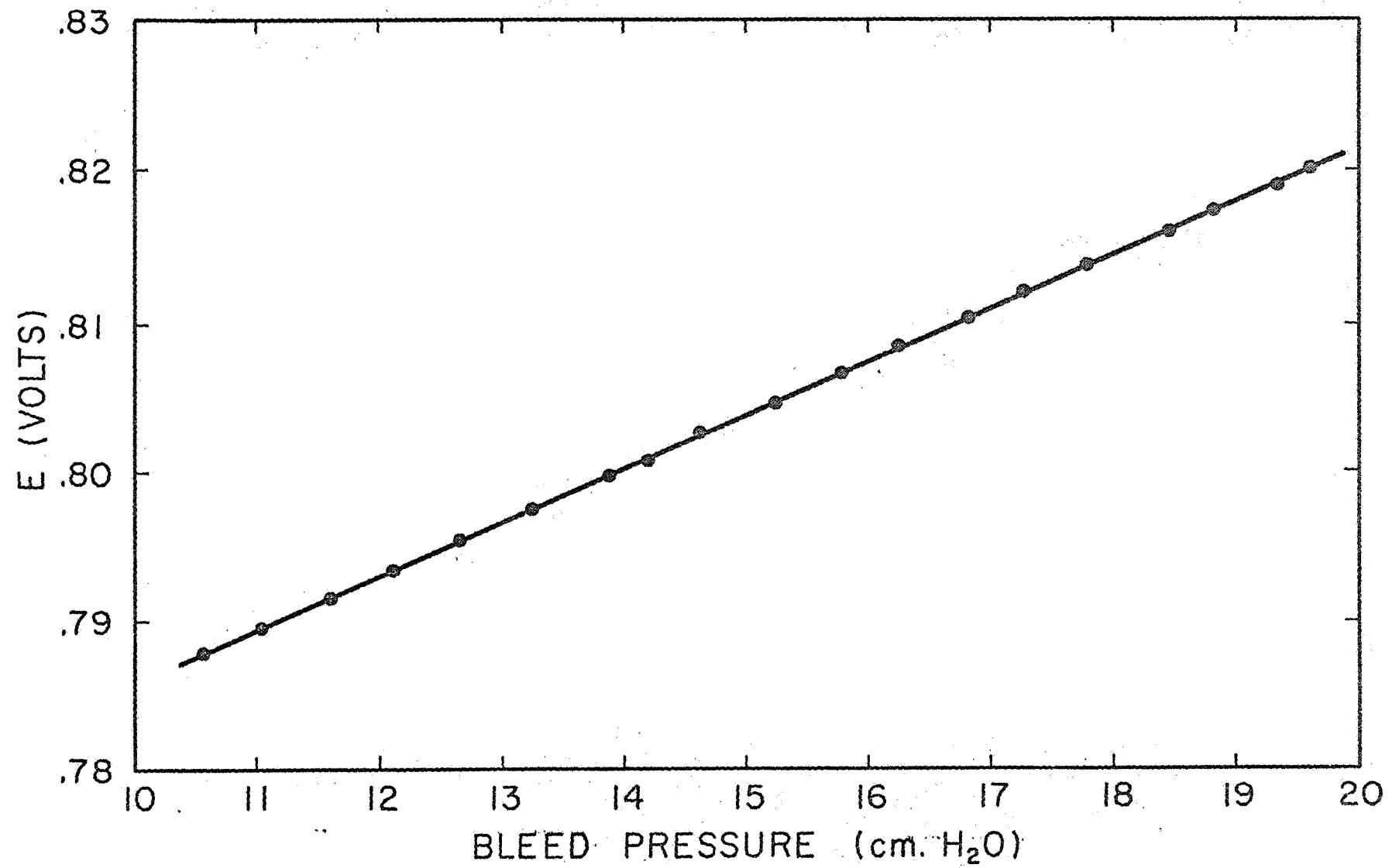


Fig. 3.4-1 Pressure Transducer Sensitivity.

a very rugged hot film sensor. The optimum operating conditions were with an air bleed back pressure of 15 cm. of water and at an overheat ratio of 1.7. At these conditions, the anemometer output was almost exactly linear as shown in Fig. 3.4-1. This figure also shows how the DC sensitivity was obtained by plotting DC output for various bleed back pressures. Typical system sensitivities (through the frequency response correction circuit) were 3.5 MV/CM H O. The sensitivities were, in practice, a function only of overheat ratio,² and consequently, were stable for long periods of time.

The system electronic and bleed flow noise was typically 0.65 MV. at 3.5 MV/CM H O sensitivity. This precluded use as a low level microphone; however, for in flow measurements this noise was not a problem. In the potential core by $x/D = 2$ corrections to intensities of less than 5% were made to compensate for this uncorrelated noise. In the mixing layer the corrections were negligible as signal-to-noise ratio was greater than 20 to 1.

The transducer was found to be an extremely rugged and versatile instrument. Successful measurements were taken at NASA Lewis Research Center in rain and snow at velocities up to 600 fps. The PT performed well.

3.4.2 Frequency Response Tests

The transducer frequency response was determined by comparing its output to the output of a calibrated 1/4 inch capacitance microphone with both instruments subjected to an acoustic field. This test was rather extensive and exacting as well as necessary each time the PT's

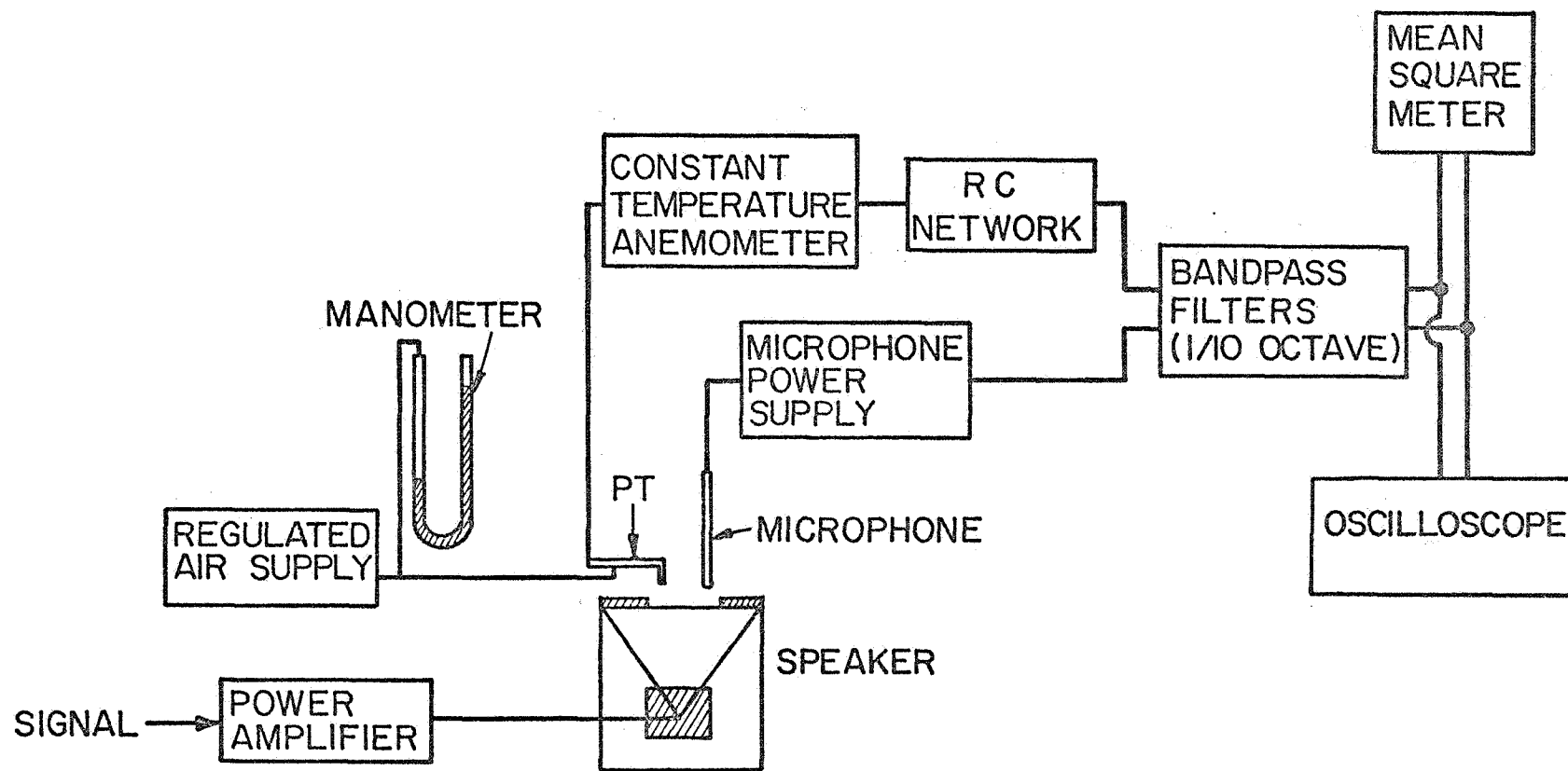


Fig. 3.4-2 Pressure Transducer Frequency Response Test.

film sensor was replaced. Therefore, both the tests and results will be described.

The equipment line-up for the test is shown in Fig. 3.4-2. The acoustic field was provided by a 14 in. diameter, loaded, diaphragm speaker for low frequencies, and a 1 in. diameter driver for high frequencies. Experiment confirmed physical intuition that interference between the PT and microphone was not a problem below 10 kHz.

Originally, it was attempted to cross correlate the microphone and PT responses to taped turbulence signals. The Fourier transformation of the cross-correlation would produce the cross-spectrum, or equivalently, the frequency response of the transducer. The results, however, were not definitive because of the large band width necessitated by the limitation of the 100-point correlator. (The correlator is described in Chapter 4.) As cross-spectral analyzers become more refined and available, this method could be more useful.

The frequency response was finally obtained by comparing the PT and microphone mean square and phase responses to a sinusoidal acoustic signal. Below 150 Hz, speaker higher harmonics were present; therefore, the responses were bandpassed through identical 1/10 octave filters. Fig. 3.4-3 shows the results of this test for signals directly from the anemometer and for those corrected by the RC network. The figure shows the corrected response to be acceptably flat and that the static sensitivity determined from varying bleed pressure is the same as the dynamic sensitivity determined from microphone comparison. The phase angles between two transducers were found to be small. In the most extreme case at high frequency, they were less than 10 degrees.

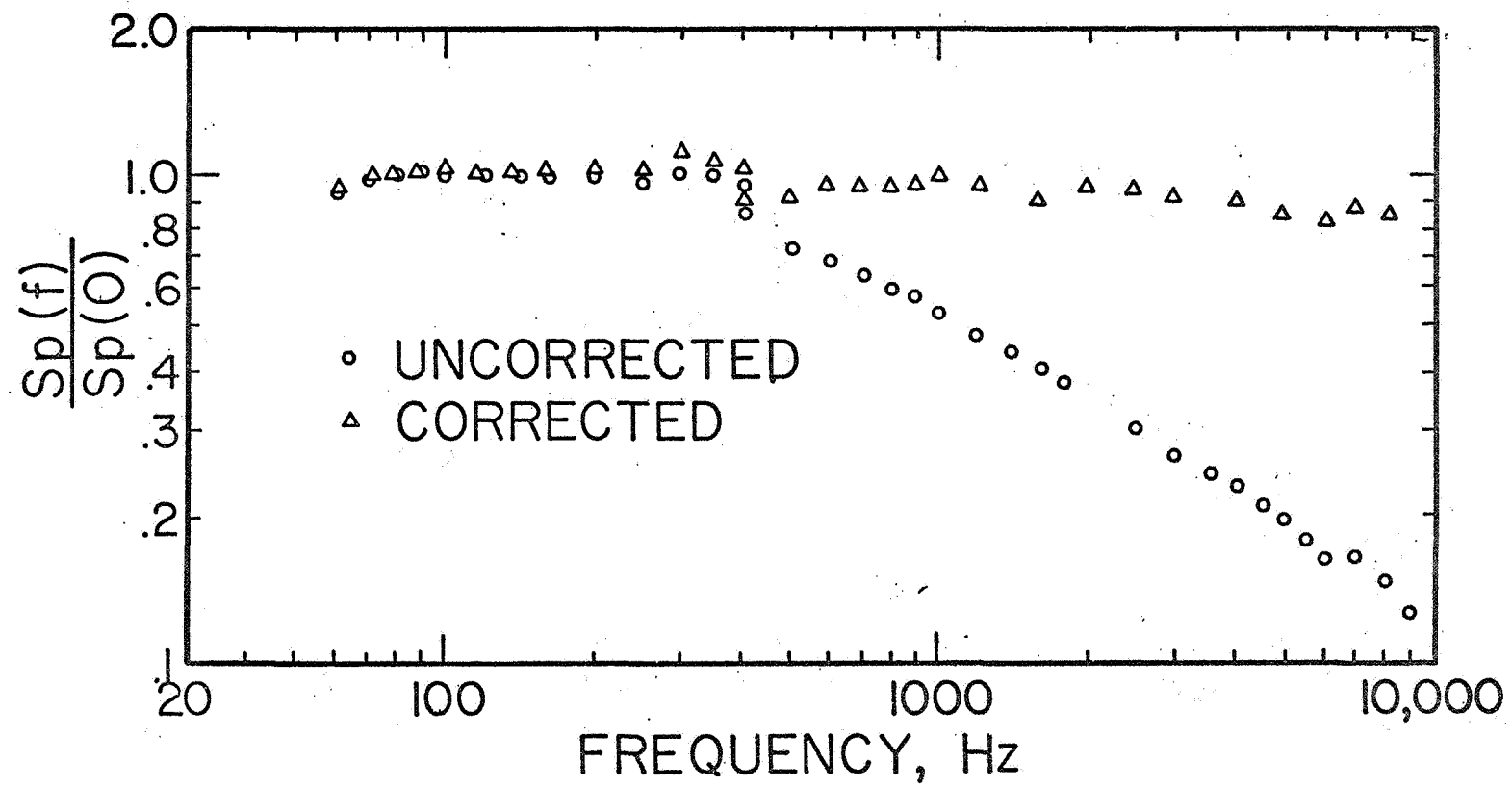


Fig. 3.4-3 Pressure Transducer Frequency Response.

Considerable uncertainty in phase angle comparison was present at high frequencies and, in practice, the correlations of pressure between two close PTs indicated no phase angle differences.

3.4.3 Velocity Error Test

The velocity error test purpose was twofold. First, the applicability of the theoretical model was verified and second, the error was evaluated from experimental input.

The applicability of the frictionless flow model and the actual sizes of A and B were evaluated in smooth steady flow near the exit plane of the jet facility. The coefficient A was found by orienting the probe for a zero yaw angle and varying the axial velocity head h_a . A zero yaw angle was found to be critical and was verified by the yaw test. Figure 3.4-4 shows the results. The pressure transducer response increases at low velocities as a result of viscous nose effects. Near 50 feet per second it becomes more gradual and by 100 feet per second an asymptotic value is attained and the coefficient A is zero.

The pressure coefficient for transverse fluctuation was evaluated by varying the normal velocity component. This was accomplished by yawing the pressure transducer in a steady flow. For this case, Eq. 3-4 reduces to

$$E(\theta) - E(0) = S_p(B-A) \frac{1}{2} \rho U_A^2 \sin^2 \theta \quad (3-10)$$

θ is the yaw angle and $E(0)$ is the voltage at zero yaw angle. The results of this test are shown in Fig. 3.4-5. The results show

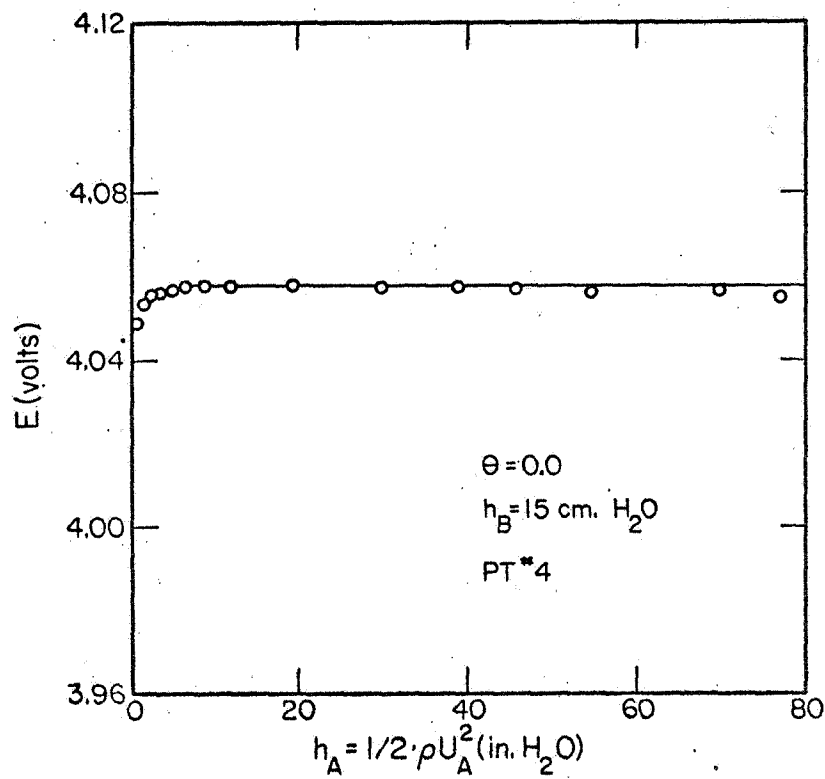


Fig. 3.4-4 Pressure Transducer Response to Axial Velocity.

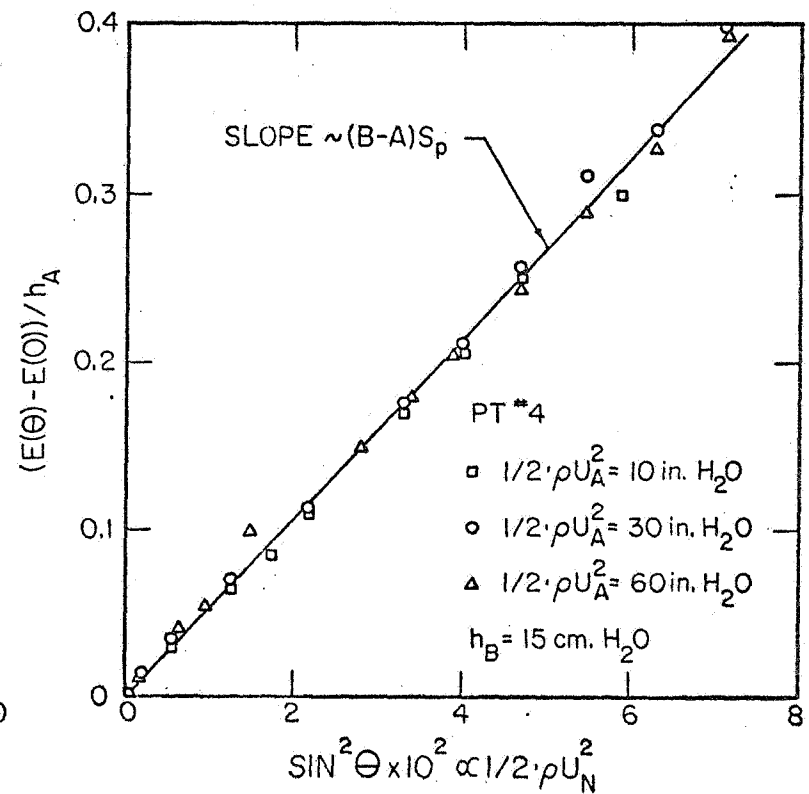


Fig. 3.4-5 Pressure Transducer Response to Transverse Velocity.

separation does not occur and that the form of Eq. 3-5 is valid for $\Theta < 16^\circ$. For greater yaw angles, the separation phenomenon was observed on an oscilloscope as a distinctive, intermittent, low-frequency phenomenon. This phenomenon was not subsequently observed in turbulent flow measurements. The value of B varied randomly from -1.1 to -0.85 for the different tests at mean velocities from 140 feet per second to 500 feet per second, showing good agreement with the theoretical value of $B = -1.0$. The variation in the experimental values is primarily from uncertainty in determining the yaw angle.

Thus, in conclusion, assuming a quasi-static response to large-scale high-speed turbulence, these tests show the ideal flow analysis describes the response of the pressure transducer to the surrounding fluctuating velocity field.

The second objective of the error tests was to calculate the velocity-induced errors from experimental information. As was previously shown, Eq. 3-8 and Eq. 3-9 predict this error in terms of A and B and an extensive set of velocity correlations. The correlation

$$\frac{\overline{V^2 \omega^2}}{\overline{V^2} \overline{\omega^2}}$$

could not be measured with an x probe and was estimated as joint normal or equal to 1.0. The other correlations were measured and are included in the Chapter 5 results.

The terms in Eq. 3-9 were evaluated and termed C_1 , C_2 , C_3 as defined in Eq. 3-11.

$$\frac{E_e^2}{E_p^2} = \frac{1}{f^2} (A^2 C_1 + A B C_2 + B^2 C_3) \quad (3-11)$$

The constants are shown in Fig. 3.4-6 for various transverse positions.

C_1 , coupled with imbalance between the pressure transducer tip and stem accelerations, produces error from axial velocity fluctuations.

A good balance, producing $A = 0.0$ is seen to be crucial near the high-velocity side. C_2 is a coupling term between axial and transverse velocity errors and is small. C_3 , with the constant B , predicts error from the unavoidable transverse fluctuating velocity. Fortunately C_3 is small and varies little across the mixing layer.

From the measurements of pressure, the factor f was estimated and a first-order estimate of pressure mean square error made. The results of this estimate, shown in Fig. 3.4-7 show the mean square error is low on the high-velocity side and increases to a maximum of about 9%. Thus, since $P_m^2 = P^2 + P_e^2$ where P_m is the measured mean square pressure, we conclude the measured pressure intensity is a maximum of 5% larger than actual.

In conclusion, an instrument suitable for fluctuating pressure measurement has been described. The instrument is small enough for two-point measurements in the jet mixing region for distances from the jet greater than 5 in. The flat frequency response, from DC to 10 kHz, is adequate for the present experiment. Ganging of the RC circuits would extend the flat range of the frequency response if required in future experiments. The sensitivity (3.5 MV/CM H₂O) and electronic noise (.65 MV) make the instrument suitable for flow measurements, but limit its use as a low pressure level sensing device.

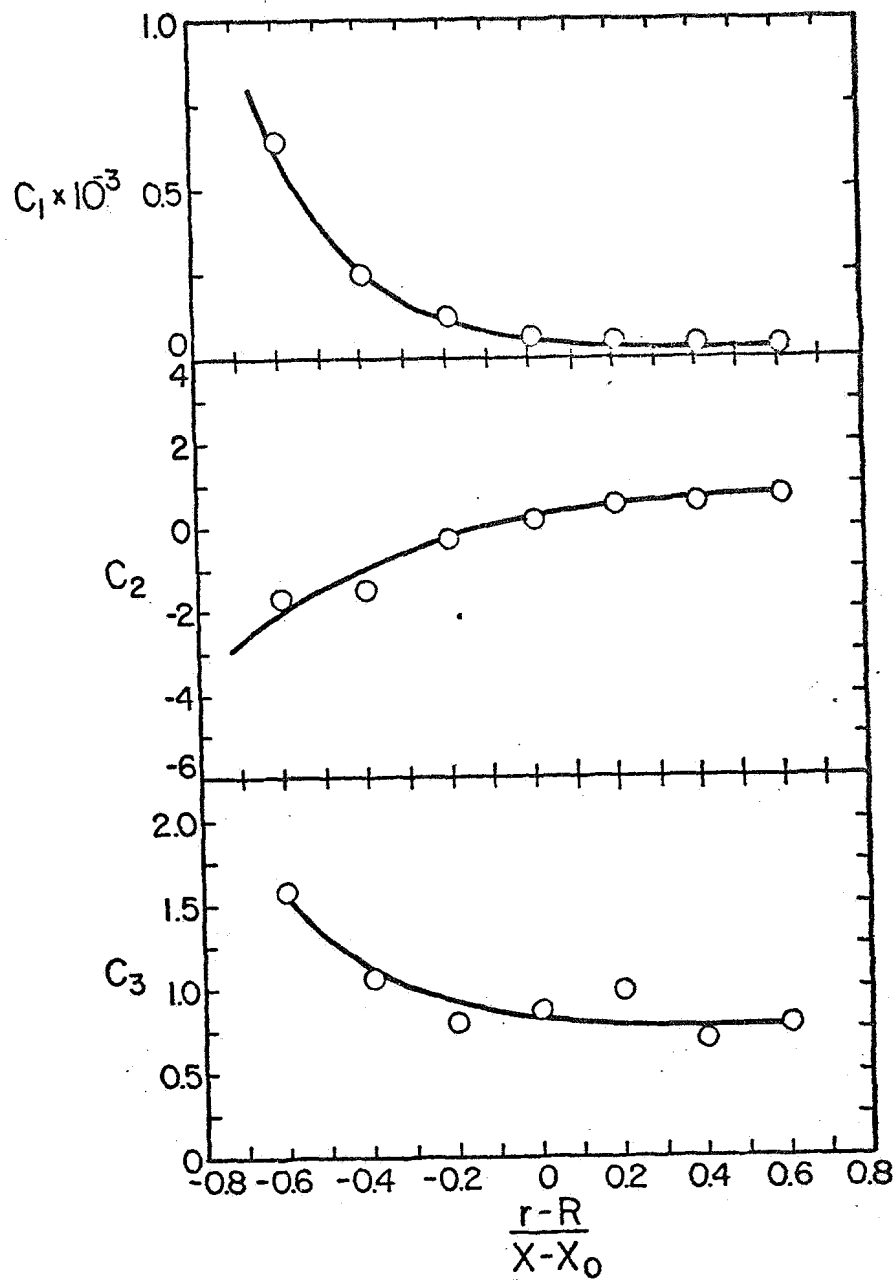


Fig. 3.4-6 Turbulence Factors Contributing to Pressure Transducer Errors. Jet Mixing Layer, $x/D = 4$, $U = 300$ ft./sec.

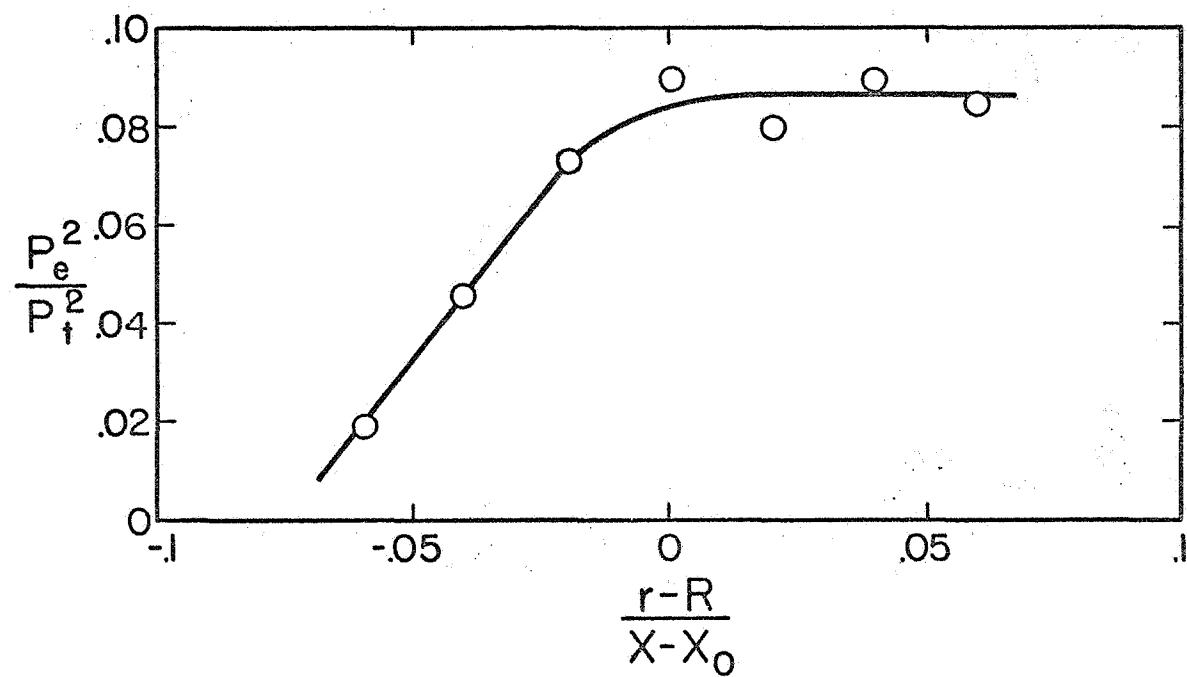


Fig. 3.4-7 Pressure Transducer Mean Square Error from Fluctuating Velocity. Jet Mixing Layer, $x/D = 4$, $U_j = 300$ ft./sec.

The theory of velocity field-induced noise shows the transducer measures intensity accurately in a shear layer at greater than 50 fps. The maximum error for mean square measurements is 9% and for intensities it is 5%. The intensity error will be shown to be acceptable in Fig. 6.1-5. It is reasonable to expect errors to space-time correlations and spectra to be bounded by the intensity errors. The model of velocity error should prove useful for evaluating the transducer's ability to measure pressure in other flows.

4. EXPERIMENT DESCRIPTION

The purpose of this chapter is to describe the flow facilities, instrumentation, and techniques used during the course of the experimental investigation. The investigation was conducted in the mixing region of a 2 1/2 in. diameter isothermal round jet. As shown in Fig. 4-1, the spatial positions of the measurements are fixed by the cylindrical coordinates x, r, θ . In standard mixing layer parlance, these coordinates are referred to as the axial, transverse, and lateral directions. As also shown in Fig. 4-1, θ is an arc length. The measurements map the mixing layer mean and fluctuating velocities and fluctuating static pressures point-by-point throughout the mixing layer. Two-point correlation measurements of velocity-to-velocity and pressure-to-pressure were made at several axial positions along the center of the mixing layer (at $r=R$) and at several transverse positions at the axial distance $x/D = 4$.

4.1 Jet Flow Facility

The experiments were conducted in a jet facility designed to exhaust a maximum of 600 standard cubic feet per minute of clean dry air. The design also provided for a flat jet efflux free from turbulent or upstream acoustic contamination. Proper sizing of the jet provided a mixing layer large enough for good resolution. Ambient and flow conditions were stable and sufficiently controllable that the experimental conditions were well defined.

A schematic diagram of the jet facility is shown in Fig. 4.1-1. Dried and filtered air is supplied through a 6 in. main line by an

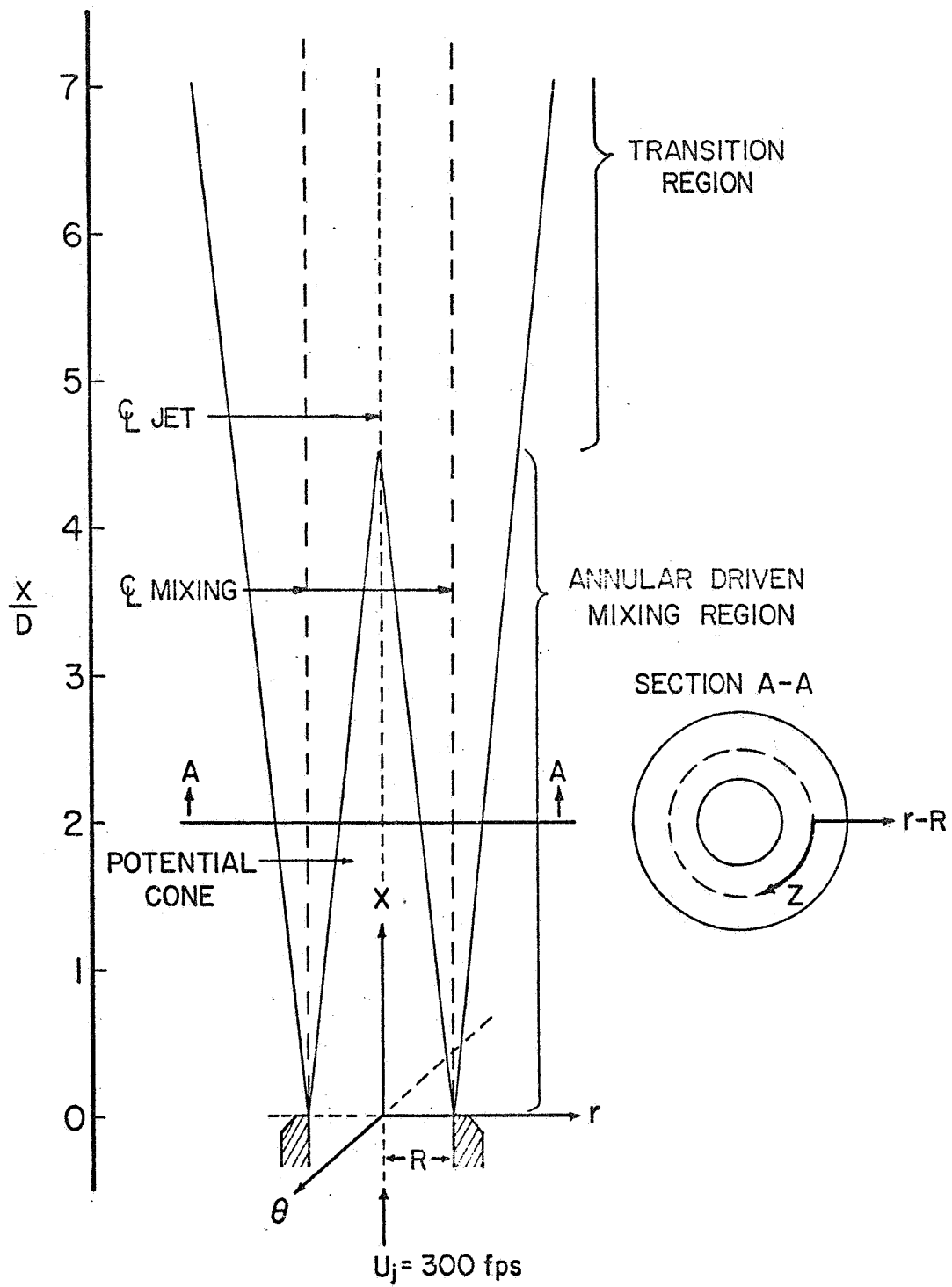


Fig. 4-1 Jet Geometry.

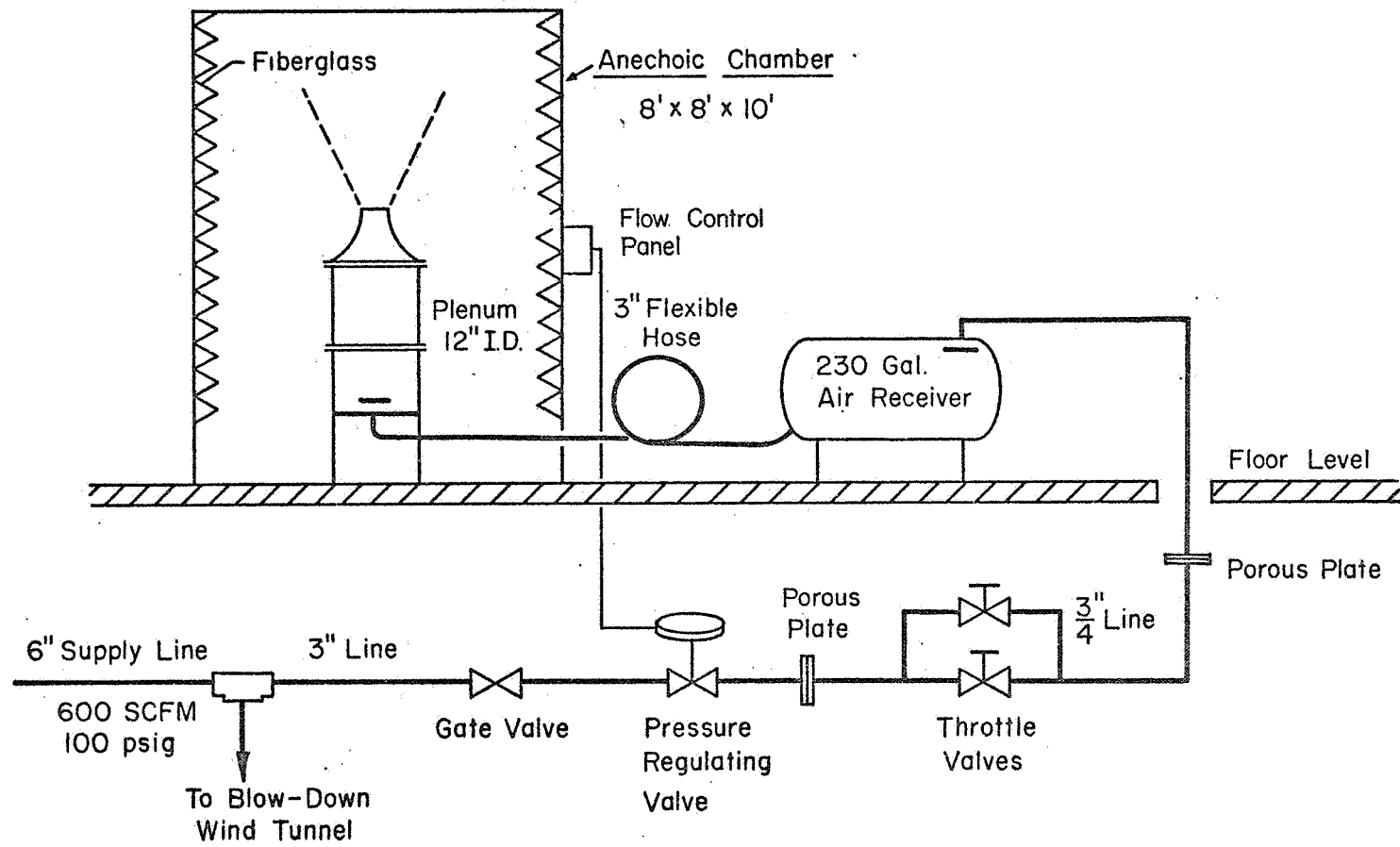


Fig. 4.1-1 Jet Flow Facility.

Ingersoll Rand Type XRE piston air compressor. The compressor rating is 600 SCFM delivered at 100 psi. A 3 in. line delivers air to the jet test facility. Pressure is regulated by a Fisher 667-ED control valve using a Type 670G remote panel loader. Porous plates having about 18% open area are located directly downstream of both the regulating valve and manifold. These are intended to help minimize the propagation of valve-generated noise downstream. On the main laboratory level the air enters a 230 gal. receiving tank. An acoustically-lined baffle plate is installed at the air inlet to prevent direct passage of sound from the entrance to the exit of the receiver. A 3 in. flexible air hose connects the receiver tank to the plenum. The hose minimizes the number of pipe fittings necessary and thereby lessens pipe-emanated noise. Additionally, the hose is looped to minimize passage of sound to the plenum.

The design of the plenum and its internals is dictated by the requirement to break-up and dissipate turbulent structure and provide flat velocity profiles to the nozzle contraction section. Dryden and Schubauer's (1947) study of reduction of wind tunnel turbulence was used as a guide.

The plenum chamber is 12 in. I.D. pipe and is 4 ft. in length. The inside surface is sandblasted and painted to ensure smooth walls. An acoustically-lined baffle is located one pipe diameter above the inlet and prevents direct passage of sound to the nozzle exit. The walls and bottom surface of the plenum are acoustically lined with fiberglass insulation material. Upon entering the plenum the air passes around the entrance baffle and is redistributed through a

porous orifice plate having 40% open area. It then, in series, passes through three canisters which provide for smooth uniform nozzle flow. The first canister contains tightly packed gauge 000 steel wool to smooth the flow. The second and third canisters each contain 4 1/2 in. long soda straws, about 1/16 in. diameter, for flow straightening. The canisters are separated by 80 mesh wire cloth and one final 80 mesh screen separates the inlet chamber from the settling chamber. The settling chamber is 30 in. long, which is sufficient to ensure viscous damping of eddies small enough to have passed through the straws and screening.

The sound absorbing walls (anechoic chamber) surrounding the jet facility greatly reduce ambient noise. The chamber measures 8 ft. square in cross section and 10 ft. in height. The bottom 2 ft. of the walls are open to permit circulation of air for entrainment. The walls are covered with corrugated 2 in. thick fiberglass insulation material for sound absorption.

The nozzle was sized to provide the largest mixing layer consistent with a reasonable exit velocity. After considering the 600 SCFM air compressor capacity, turbulent velocity scale sizes compared to sensor dimensions, and the possibility of increasing pressure transducer errors for lower jet speeds, the 2 1/2 in. diameter jet was selected. This choice provides a large enough mixing layer that two sensor measurements were possible at an axial distance greater than two diameters. This is important because it allows measuring axial two-point variations from the two diameters to the end of the driven mixing layer at five diameters. The 300 fps maximum jet velocity was

sufficient to ensure small pressure transducer errors. However, it did restrict the experiment with the 2 1/2 in. nozzle to the incompressible range.

The contour of the jet nozzle was selected to provide flat velocity profiles and thin boundary layers. Smith and Wang (1944) have shown that a nozzle contour following the axisymmetric stream function from inlet to outlet preserves an entering flat velocity profile. The coordinates for the nozzle were taken from their Table 1 curve "b". This profile predicts uniformity of throat speed, with negligible boundary layer, within 0.2 percent. The contraction ratio from the plenum to exit plane is greater than 22:1 and thus produces very thin boundary layers and small scale low intensity turbulence as shown in Chapter 5. The inside nozzle contour merges squarely with the nozzle lip at the exit plane. The lip is 1/8 in. wide and its outside edge is beveled at 45 degrees.

The traversing mechanism was designed to hold two probes rigidly and accurately, minimize flow disturbance and allow probe separation in any direction. As shown in Fig. 4.1-2, the stationary probe can be positioned radially and axially. The relative probe can be traversed with respect to the stationary probe in any of the three orthogonal directions. The three traversing axes for the relative probe motion may be tilted relative to the jet axis to accommodate axial separations along lines of constant mean velocity or to do yaw tests.

The sensor position was fixed within .02 in. with a bull's eye at the jet exit plane and then traversed to the measuring position. Verniers on the traversing devices were used to position the sensors to

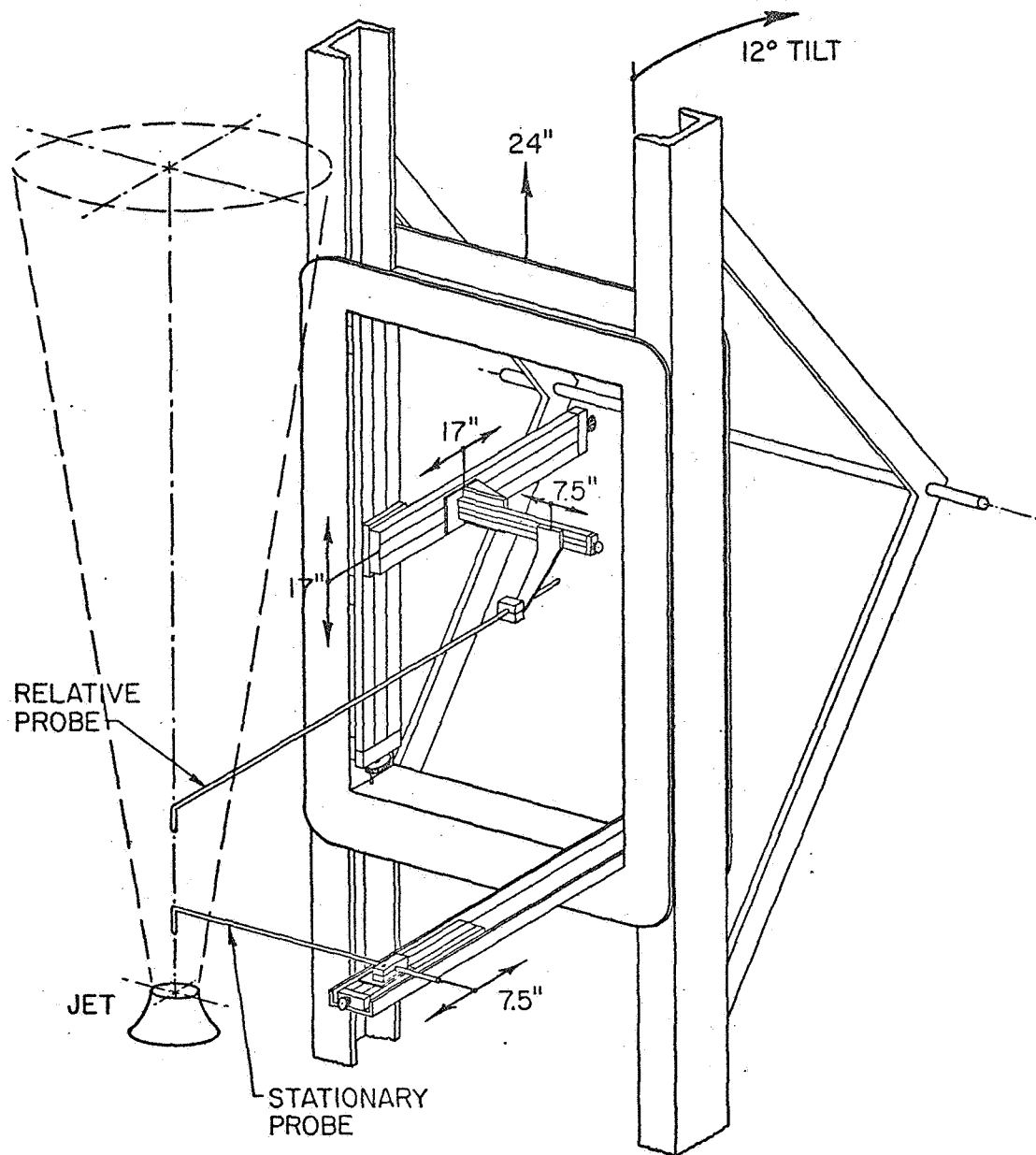


Fig. 4.1-2 Traversing Mechanism.

± 0.01 in. This uncertainty fixes the errors in probe separation. The jet and traversing mechanism were leveled so that the jet centerline and apparent traversing centerline deviated less than ± 0.01 in./axial foot.

The flow facility is instrumented with a bank of manometers; a flow thermocouple, and barometer to measure flow velocities. The manometer bank consists of a five-inch inclined manometer and 30-inch and 60-inch U tube manometers. At the experimental conditions the uncertainty in jet stagnation pressure is about 0.1 inch of water. The iron-constantan thermocouple measured stagnation temperature to $\pm 0.2^\circ$ F. Although not a large influence, varying barometric pressure of ± 0.005 in. Hg, was recorded and used to compute jet exit velocity.

To estimate the errors introduced in the mean velocity, we consider the one-dimensional, steady-flow treatment of isentropic flow as discussed by Shapiro (1953) and summarized by Dean (1953). Using the steady flow equations for reversible adiabatic flow with no shaft work, and the relations between pressure, temperature, and density for an isentropic process of a perfect gas, it may be shown that:

$$V^2 = \frac{2c^2}{k-1} \left[\left(\frac{p_e}{p_o} \right)^{\frac{k-1}{k}} - 1 \right] \quad (4-1)$$

where V is the stream velocity, c is the speed of sound at the jet exit, p_o is the stagnation pressure, p_e is the stream exit pressure, and k the ratio of specific heats. Using the perfect gas relations,

we have:

$$c = \left(\frac{\partial p}{\partial P} \right)^{1/2} = (kRT)^{1/2} \quad (4-2)$$

where R is the gas constant and T is the temperature. Using the relations, we can determine the error in jet velocity from the errors in p_0 , p_e , and T . Using $T = 80^\circ \text{ F}$ (540° R), $p_e = 30 \text{ in. Hg}$, $p_0 = p_e + 20 \text{ in. Hg}$ as the standard operating conditions, we may show that the errors introduced are 1.47 ft/sec./ $^\circ \text{ F}$ for temperature, 5.87 ft/sec./in. Hg in stagnation pressure, and 83.7 ft/sec./in. Hg. Using the previously given error estimates, the mean velocity was accurate to 1.3 ft/sec.

The operating characteristics of the jet facility were as expected from the previous design comments. The exiting jet was free from turbulence and acoustic contamination and the mean velocity profiles were flat. These important characteristics are documented in Chapter 5. Stagnation temperature and barometric pressure were quite stable. Operating at full air compressor capacity sometimes caused low frequency (0.5 cycle per minute) oscillations of jet exit velocity and an inability to attain exact flow rates. The oscillations were damped by delicate manipulations of the flow control valve. Accuracy was ensured by reading stagnation pressure along with each point and discarding points further than 5% from the selected speed. The analysis accounted for non-constant exit velocity so that the effect of this difficulty on the results is negligible.

4.2 Measurement Instrumentation and Techniques

4.2.1 Instrumentation

Both the velocity and pressure measurements were made with standard commercially available hot-wire anemometry and auxiliary electronic equipment. The measurement system layout for single-point measurements is shown in Fig. 4.2-1, and for two-point measurements in Fig. 4.2-2. These specific layouts are for the more complicated x probe measurements. Single wire measurements used half the system shown. The pressure measurements used a similar system except the linearizers were replaced with the frequency compensation network discussed in Chapter 3.

The anemometers shown are matched DISA 55D01 models and were operated in the constant temperature mode at an overheat ratio of 1.6 with the pressure transducer and 1.8 for hot-wire sensors. The velocity measurements used DISA 55D10 linearizers. The linearization constant and voltage offset were selected from anemometer bridge output data in a manner similar to that described in the unit operating manual. Filters, numbers 1, 2, 3 and 4, are a matched bank of BAY LAB 5123 low-noise amplifiers. The cutoff at 18 dB/octave above 30 kHz eliminated electronic noise and did not detectably alter the turbulence signal. Prior to recording, the signals were AC coupled with a TSI 1015C analog correlator. This allowed full AC-signal resolution on the tape. The long time average of DC voltages were obtained by passing the full signal through two Kron-Hite 3750 filters. The filters were operated in the low pass mode with a 24 dB/octave cutoff above a nominal 2 Hz. The cutoff frequency was decreased for

SINGLE POINT MEASUREMENT INSTRUMENTATION

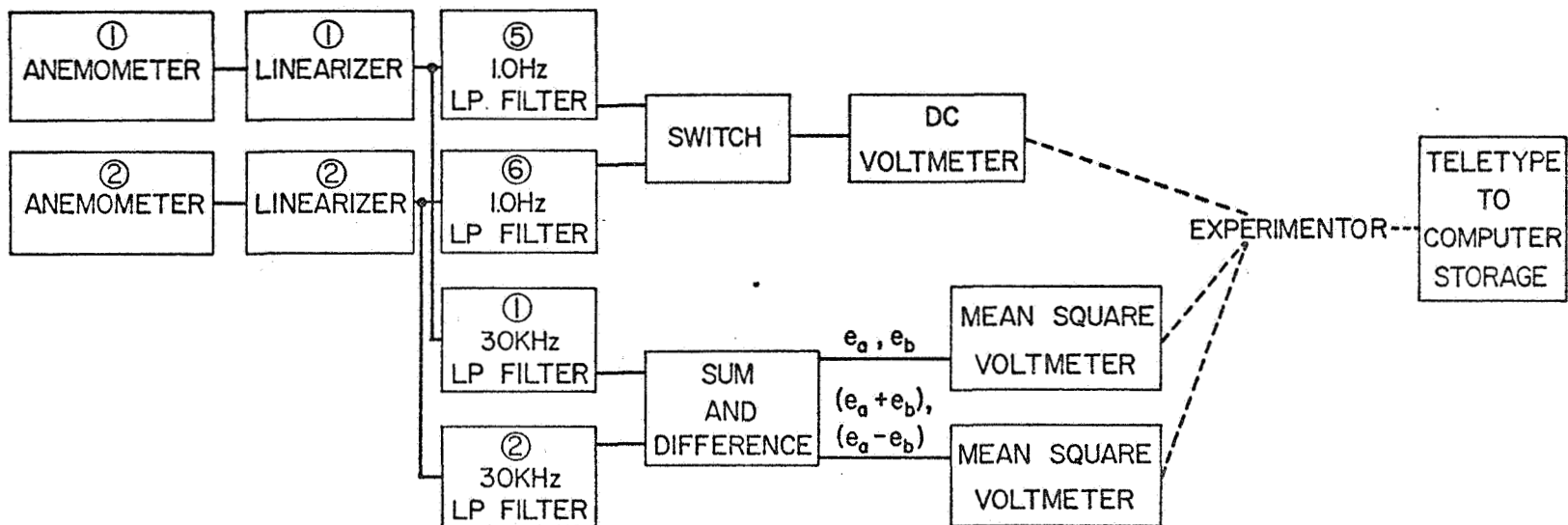


Fig. 4.2-1 Single Point Measurement System.

MEASUREMENT AND RECORDING INSTRUMENTATION

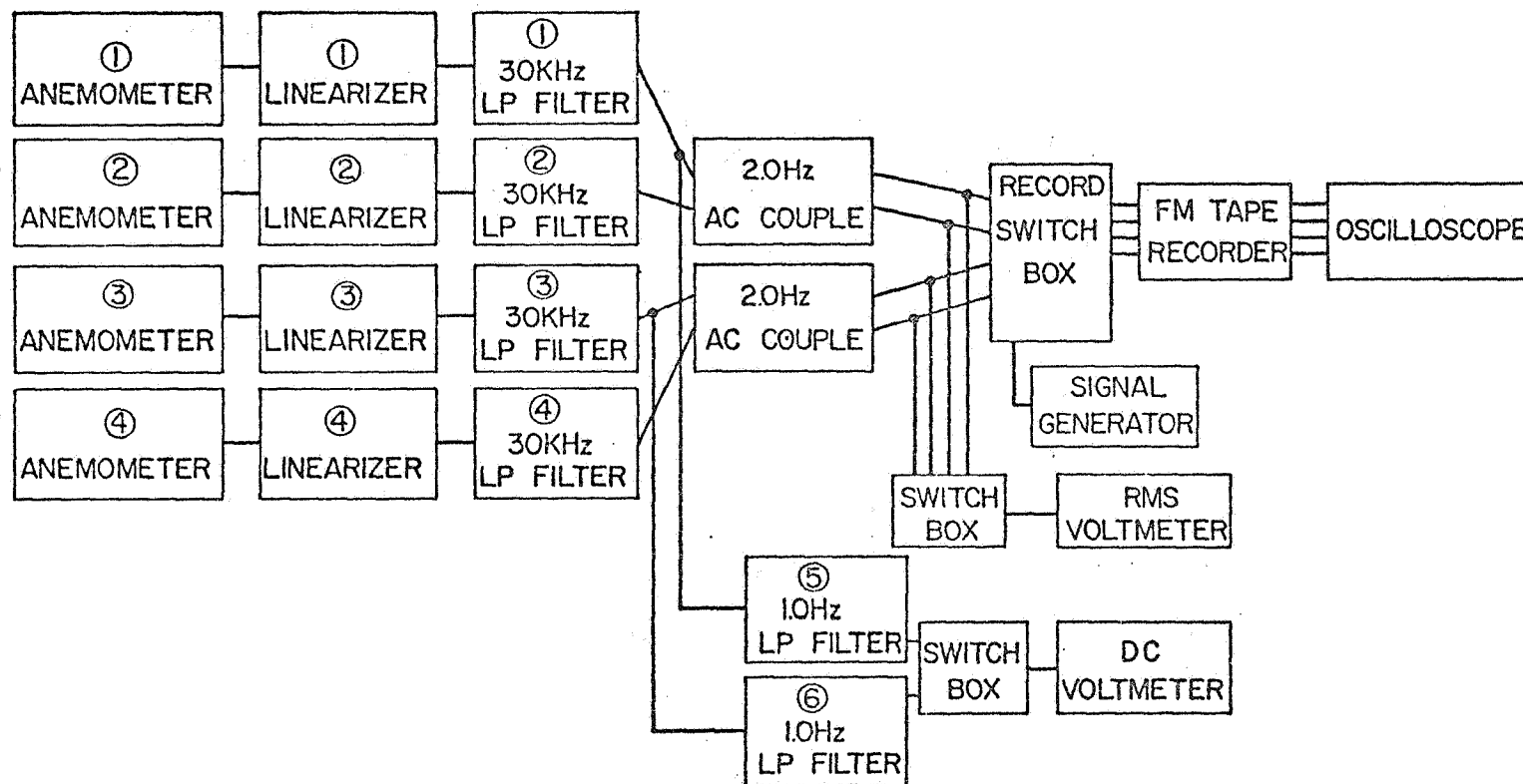


Fig. 4.2-2 Two Point Measurement System.

lower frequency turbulence so that DC voltage fluctuations were less than 1%. The DC voltages were read on a digital voltmeter. The signals were recorded on a Sangamo 3564 14-track, frequency modulated magnetic tape recorder. It was operated at 60 ips to obtain a bandwidth of d.c. to 20 kHz. The single-point readings were transmitted to the University IBM 360-75 computer for storage via a timesharing teletype and acoustic telephone link (in local terminology, the PLORTS system). When the timesharing system load was light and operation was fast and reliable, the readings were stored directly. At other times, the data was recorded on paper tape and later filed into computer storage. The overall system operated accurately and reliably. The matched anemometry and electronic channels prevented phase shift differences. Hot wire and pressure transducer sensitivity and square wave responses were measured through the whole system to eliminate an accumulation of gain errors and to ensure no phase shifts. After the initial shakedown the system electronics were kept energized. This provided remarkable stability and reliability.

4.2.2 Hot-Wire Sensors

Both single hot-wire sensors and two hot-wire x-configuration sensors were used during the investigation. The single sensors were used for preliminary work and a limited investigation near the jet exit. The single wire probes were standard TSI 1210-T1.5 models with tungsten 0.00015 in. diameter sensors.

The single-point final measurements were made with a TSI 1241-T1.5 end flow x probe. The two-point measurements were made with a

TSI 1240-T1.5 cross flow x probe upstream and the TSI 1241-T1.5 end flow x probe downstream. The probe arrangement for these measurements is shown in Fig. 4.1-2. Both the end flow x and cross flow x utilize 0.00015 in. diameter tungsten sensors. The sensor length to diameter ratio is 333.

The support prongs of the cross flow x probe were necessarily normal to the mean flow direction. With this arrangement in smooth flow, some shedding from the prongs was noticed as a very high frequency sensor response. Although the cause of some initial concern, this unwanted disturbance was eliminated by the use of the 20 kHz low pass filter. In a final check, it was found that the axial and transverse velocity intensities and shear stress from the cross x probe and end flow x probe measurements agreed closely.

The response equations for the wires follow the assumption that only normal components of velocity have significant cooling effects, the sine law cooling hypothesis. Champagne (1965) determined the validity of such a hypothesis for $\ell/d > 300$, where ℓ is the active wire length and d is its diameter. Since our x probes are within this range and our yaw tests confirmed normal cooling; we have used sine law response in data analysis.

Spencer (1970) has shown that measurements with a three-sensor probe analyzed with a sine law cooling response equation are practically exact. An application of his method to a two-sensor x probe cannot be exact because a third component of velocity cannot be precisely accounted for. His general response equations, however, indicate little error from neglecting the influence of the lateral component of

velocity in x probe measurements. Because of the complexity of two-point, three sensor probe measurements and because of the larger size of the three sensor probe, the less exact but certainly adequate x probes were used.

The sine law cooling response equations for an x probe oriented with the plane of the x in the $x-r$ plane are:

$$\begin{aligned} E_1 &= E_1^0 + S_1' \sin \alpha_1 (U - c \tan \alpha_1 V) \\ E_2 &= E_2^0 + S_2' \sin \alpha_2 (U + c \tan \alpha_2 V) \end{aligned} \quad (4-3)$$

The sensitivity of the hot wires to normal velocity is S (volts/ft/sec.). The angle between the wire and mean flow direction x is α as shown in Fig. 4.2-3. E^0 is a DC offset of the linearized response.

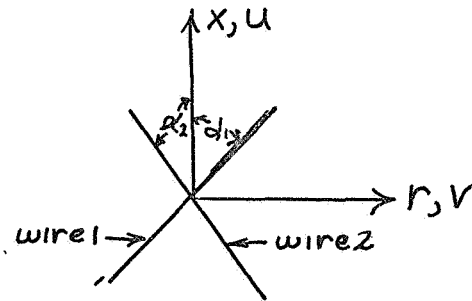


Fig. 4.2-3 X-Probe Geometry.

The sensitivities and angles of inclination of the wires were nominally matched although they were carefully measured and accounted for in data reduction. The sensitivities were determined by placing the probe in smooth flow near the jet exit plane and varying the mean velocity. Since V is zero during this calibration the constants determined in Eq. 4-3 are $S \sin \alpha$ and E^0 . The constants were determined from a least

squares data fit. A typical uncertainty for a 95% confidence interval¹ in sensitivity determination was 1%.

The sensor inclination was determined from standard yaw tests. These tests also verified sine law cooling. In smooth flow the response equation may be written as:

$$E(\alpha, \phi) = S \sin(\alpha + \phi) U \quad (4-4)$$

where α is the fixed wire orientation and ϕ is the yaw angle. Thus, we may write:

$$f(\alpha, \phi) = \frac{E(\alpha, \phi)}{E(\alpha)} = \frac{\sin(\alpha + \phi)}{\sin \alpha} = \cos \phi + \cot \alpha \sin \phi \quad (4-5)$$

For small angles $\sin \phi \approx \phi$ and $\cos \phi \approx 1$ thus:

$$f(\alpha, \phi) = 1 + (\cot \alpha) \phi \quad ; \quad \phi \sim 0 \quad (4-6)$$

The yaw angle was determined from Eq. 4-6 and verified along with sine law cooling by plots of Eq. 4-5.

Estimates of error in this determination are difficult because of the trigonometric form of the yaw response. However, an upper bound, including experimental scatter, departures from sine law cooling and the small angle approximation can be estimated as the least squares uncertainty from a data fit in Eq. 4-6. This upper bound of the uncertainty for 95% confidence interval is 10%. From the plotted data

1

Li (1964) gives a physical interpretation of least squares in terms of linear regression theory. This allows estimation of slope and offset and the variances of the estimates.

it is obvious that this large error estimate is due primarily from the small angle approximation and that the uncertainty in a 95% confidence interval is easily less than 5%.

The x probes were offset in the lateral direction to avoid upstream probe interference. The optimum offset was selected from a grid of measurements in the x - z plane as the minimum lateral separation for no noticeable mutual interference. In the turbulent mixing layer no interference was discernible on the oscilloscope or in RMS values even for very small separations. The strength of the mixing layer turbulence completely dominated any interference or upstream sensor shedding. To be conservative, however, the test was repeated in smooth flow where upstream shedding was observed on the downstream sensor for lateral separations less than 0.075 in.--a distance only slightly larger than the x probe support-to-support distance. It was decided to be consistently conservative and use this offset ($\epsilon_z = .075$ in.) for all the two-point measurements. In retrospect, this conservatism was unnecessary for transverse separations. Because spatial separability was not valid for velocity correlations, the offset was not easily accounted for.

4.2.3 Pressure Transducer

The pressure transducer operating characteristics and errors are discussed in detail in Chapter 3. The general operation of the instrument in this experiment will be discussed here.

The placement of the sensors in the flow were similar to that described for the x probes and shown in Fig. 4.1-2. Great care was taken to align the probe directly into the flow to minimize error

from axial velocity fluctuations. The initial alignment was made by visual comparison to a plumb line and checked by a yaw test described in Chapter 3.

The transducers were offset in the lateral direction as were the x probes. The guiding philosophy was to minimize offset for no noticeable disturbance for any separation. To find this optimum offset, a series of measurements were made in the planes $(\Delta x, \Delta \theta, \Delta r = 0)$ and $(\Delta x = 0, \Delta \theta, \Delta r)$. Correlation curves between the two sensors, as well as curves of the mean square signal of each sensor, were developed. It was found for $\Delta \theta > 0.15$ in., no extraneous signals were present on either sensor and that the correlations were smooth and believable. Further, preliminary indications of space separability of correlations was found. This indeed proved to be valid, as explained in Chapter 6, and it provides a method for deducing true correlations from offset measurements. Hence, it was decided to be very consistent and use a lateral offset of $\epsilon_z = 0.15$ in. for all the two-point pressure correlation work.

The response of the pressure transducers was linear and flat in frequency as discussed in detail in Chapter 3. Thus, the response equation was very simple:

$$(\overline{p^2})^{1/2} = \frac{1}{S_p} (\overline{e^2})^{1/2} \quad (4-7)$$

where S_p (volts/CmH₂O) is the sensitivity determined by the DC response to varying bleed pressures.

4.3 Data Analysis Systems and Techniques

4.3.1 Point Intensity and Mean Distributions

The analysis of pressure and velocity data follows directly from the instruments' response in Eq. 4-3 and Eq. 4.7. The pressure analysis was particularly simple. Pressure intensities and the jet stagnation pressure for each measurement position were processed directly into computer storage files as shown schematically in Fig. 4.2-1. The pressure analysis program based on Eq. analyzed the data and stored the results for subsequent output or analysis.

The velocity analysis determined the axial and transverse intensities, shear stress from inputs of the AC-coupled mean square of each wire output, and the mean square of their difference. The mean velocities were computed from the DC response. Equation 4-3 is the basis of the matrix response equations below for fluctuating velocities:

$$\begin{pmatrix} \overline{e_1^2} \\ \overline{e_2^2} \\ \overline{(e_1 - e_2)^2} \end{pmatrix} = \begin{pmatrix} S_1^2 \sin^2 \alpha_1 & 2S_1^2 \sin \alpha_1 \cos \alpha_1 & S_1^2 \cos^2 \alpha_1 \\ S_2^2 \sin^2 \alpha_2 & -2S_2^2 \sin \alpha_2 \cos \alpha_2 & S_2^2 \cos^2 \alpha_2 \\ (S_1 \sin \alpha_1 & 2(S_1 \sin \alpha_1 - S_2 \sin \alpha_2) & (S_1 \cos \alpha_1 \\ -S_2 \sin \alpha_2)^2 & (S_1 \cos \alpha_1 + S_2 \cos \alpha_2) & +S_2 \cos \alpha_2)^2 \end{pmatrix} \begin{pmatrix} \overline{U_1^2} \\ \overline{U_1 V_1} \\ \overline{V_1^2} \end{pmatrix} \quad (4-8)$$

A velocity analysis program took the sensitivities and wire angles and the data file containing reading voltages and solved the above set of equations for the velocity intensities and shear stress. A similar equation was solved for the mean velocities. As with pressure the results were filed for further analysis.

4.3.2 Two-Point Correlations

4.3.2.1 Velocity Signal Generation

As indicated earlier, the x probe signals were FM recorded without sum or differencing to attain a signal directly proportional to a specific wire response. Upon playback, the signals were AC coupled and, by adjusting the relative gain between the two signals on each probe, a specific velocity component could be generated. For example, the two-wire response equations could be written as:

$$\begin{aligned} E_1 &= S'_1(U_1 + ctn\alpha_1 U_2) \\ E_2 &= S'_2(U_1 - ctn\alpha_2 U_2) \end{aligned} \quad (4-9)$$

After applying a gain factor to each signal, the signals were summed to give:

$$\begin{aligned} E &= G_a E_1 + G_b E_2 \\ \text{or} \quad E &= (G_a S'_1 + G_b S'_2) U_1 + (G_a S'_1 ctn\alpha_1 - G_b S'_2 ctn\alpha_2) U_2 \end{aligned} \quad (4-10)$$

where G_a and G_b are the gain factors. By choosing the gain factors in a ratio such that the coefficient of U_2 would vanish, a signal proportional to U_1 was generated. A similar procedure involving the difference of the two raw signals generated a signal proportional to U_2 .

4.3.2.2 Correlation

The appropriate signals were then fed to a TSI 1065A digital correlation and probability analyzer to generate the appropriate auto and

cross correlations. The signals were AC-coupled and the damping time, chosen to satisfy the Nyquist sampling criteria, was set at 50 μ sec., corresponding to an upper frequency limit of 10 kHz. The correlator samples and digitizes the raw signal with an accuracy of 1 part in 256. The number of lagged product type sums is adjustable, being a multiple of 1024, normally taken as 128 x 1024. The total amount of real time data analyzed is this product times the lag time, chosen as 50 sec. This corresponded to 6.55 sec. of real time data for each sample co-variance. The correlator has the ability to resume this summation process, leading to an effective averaging of sample co-variances. Smooth auto co-variances were produced by averaging three such sample co-variances, while cross co-variances tended to require between three and six sample co-variances.

The co-variances were then normalized and plotted on a Houston 2000 plotter. In addition to this visual output, the co-variances were punched out on binary tape, in blocks of 100 points, corresponding to the 100 bin memory capability of the correlator. These binary tapes were read and converted to BCD form by either a Lockheed Sue or a PDP-12 digital computer, and then stored on disk files on the IBM 360-75, for subsequent digital analysis. A system diagram is shown in Fig. 4.3-1.

Since the co-variances so produced were from signals only proportional to the velocities, a normalization procedure was employed

CORRELATION ANALYSIS INSTRUMENTATION

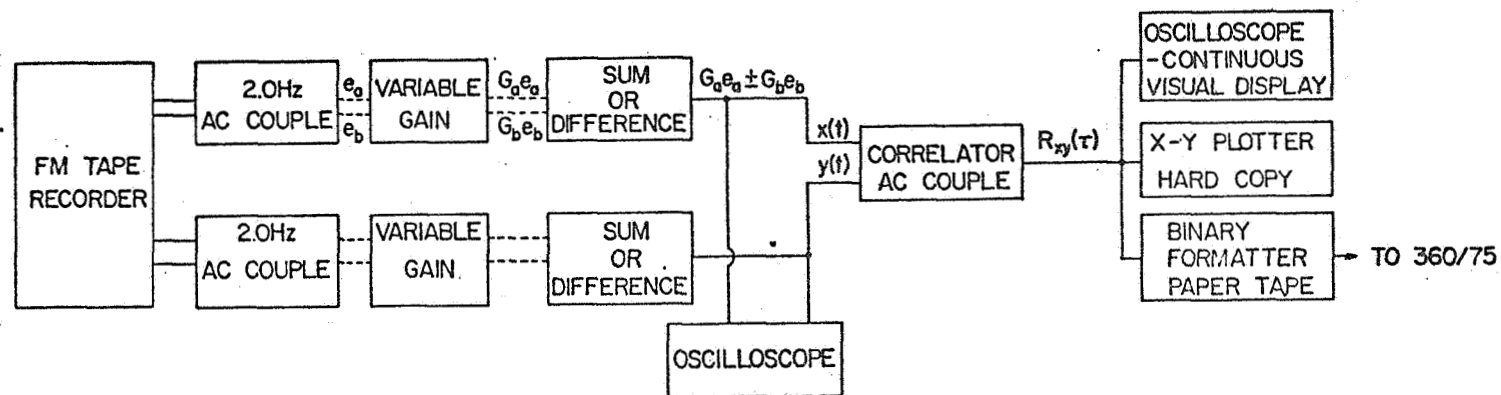


Fig. 4.3-1 Correlation Analysis System.

to generate the appropriate correlation function. The correlator produces an output like:

$$C_{xy}(\tau) = \overline{(K_1 X(t))(K_2 Y(t+\tau))}$$

where K_1 and K_2 are correlator operational constants. By running auto-correlations on both X and Y , we get:

$$C_{xx}(\tau) = K_1^2 \overline{X(t) X(t+\tau)}$$

and

$$C_{yy}(\tau) = K_2^2 \overline{Y(t) Y(t+\tau)}$$

We then determined the normalized correlation from:

$$R_{xy}(\tau) = \frac{C_{xy}(\tau)}{(C_{xx}(0) C_{yy}(0))^{1/2}}$$

4.3.2.3 Errors in Correlation

Bendat and Piersal have discussed errors in estimates of means, variances, correlations and spectra using a model of bandwidth limited white noise to show that normalized errors go as $1/BT$ where B is the bandwidth and T is the integration time. For the turbulence signals of interest, the integration time was generally greater than 20 seconds, while the bandwidth was at least 1 kHz. Numbers in this range give very small normalized errors and although the white noise model is not

exactly appropriate, it gives an indication of approximate errors. In general, the correlations were run long enough to give a smooth appearance to the correlator output. That is, the necessary averaging time was estimated experimentally.

The correlator is designed to handle signals with a full-scale input of 100 *mv RMS* (sine wave). Signals larger than this are attenuated by adjusted front panel controls. By adjusting these controls, various size signals could be analyzed. When the attenuators were set too high, resolution of the signal was not good. When the attenuators were set too low, the peaks of the signal were clipped off. The following procedure was used to maximize benefits.

Since the instrument was capable of producing probability density plots, such analyses were made. Once again, the attenuator setting determined the range of allowable signals. The density is centered about the center bin, bin(50), with negative and positive deviations in bins 0 to 49 and 51 to 99, respectively. It can be shown that if a signal is Gaussianly distributed, the ratio of either bin(0) or bin(50) is a direct indication of the probability that signals above a certain level were clipped. The attenuators were actually set so that $\text{bin}(99)/\text{bin}(50) = 0.25$, which would indicate that had the signals been Gaussianly distributed, less than 1% of the signal would have been eliminated.

5. VELOCITY FIELD RESULTS AND DISCUSSION

The explicit relationship between the turbulent pressure and velocities was developed in Chapter 2. This dependence outlines the thrust of this section--to experimentally describe and interpret pressure-determining features of the turbulent velocity field. Additional data will demonstrate that the flow facility is nearly "standard", allowing comparisons with various other investigators. The results are presented in a conventional non-dimensional form consistent with a similarity hypothesis. That is, turbulent velocities scaled by jet exit velocity U_j , are plotted as a function of dimensionless position $\eta = \frac{r-R}{x-x_0}$.

5.1 Nozzle Exit Conditions

To allow detailed comparisons with other mixing layer experiments, it can be important that the initial conditions of the flow systems be matched. This view supplements and adds detail to the traditional idea that free shear flows are largely self preserving and hence a function only of the flow system dynamics and external constraints. The important influence of initial conditions upon the approach to similarity was discussed by Bradshaw (1966). Crow and Champagne (1971) showed that large changes in the mixing layer structure could be brought about by low level jet exit velocity surging. Even though a study of initial conditions is not the objective, we conclude that the jet should be experimentally standard and the jet exit conditions should be well documented. The experimental standard for jet studies is an ideal nozzle which provides a flat mean velocity profile that is free of

turbulence and acoustic contamination and which merges discontinuously with infinite quiescent surroundings. The design of the flow systems and nozzle to approach this ideal was presented in Chapter 4. To evaluate our system, a series of tests was run primarily at, or near, the nozzle exit plane.

The mean and fluctuating velocity profiles were measured .033 inches downstream of the exit plane. As shown in Fig. 5.1-1, the mean velocity profile is flat. The boundary layer thickness, displacement thickness, and momentum thickness were found to be $\delta = .018$ in., $\delta^* = .0028$ in., and $\Theta = .0001$ in., respectively, so small that measurement and computation uncertainties could produce errors that are a large fraction of δ^* and Θ . The small values of δ^* and Θ however, show that the gradual rise in $\bar{u}/u_j$, as the boundary layer is approached, very nearly compensates for the momentum deficit of the boundary layer.

The background turbulence intensity was so low that it could not be distinguished from the anemometer system noise. The level of .1 to .2% shown in Fig. 5.1-1 consequently should be an upper bound for turbulence and acoustic contamination. The level of acoustic contamination is shown to be small by assuming $p' \sim \frac{1}{2} \rho \bar{u}^2$. Then

$\frac{p'}{\sqrt{2} \rho \bar{u}^2} = (.002)^2$. The acoustic sound power level* is about 40 dB referenced to .002 μ bars. These values are well below pressure values

*Sound Power Level is defined as $SPL = 10 \log_{10} \frac{P'}{P_{ref}}$ 2

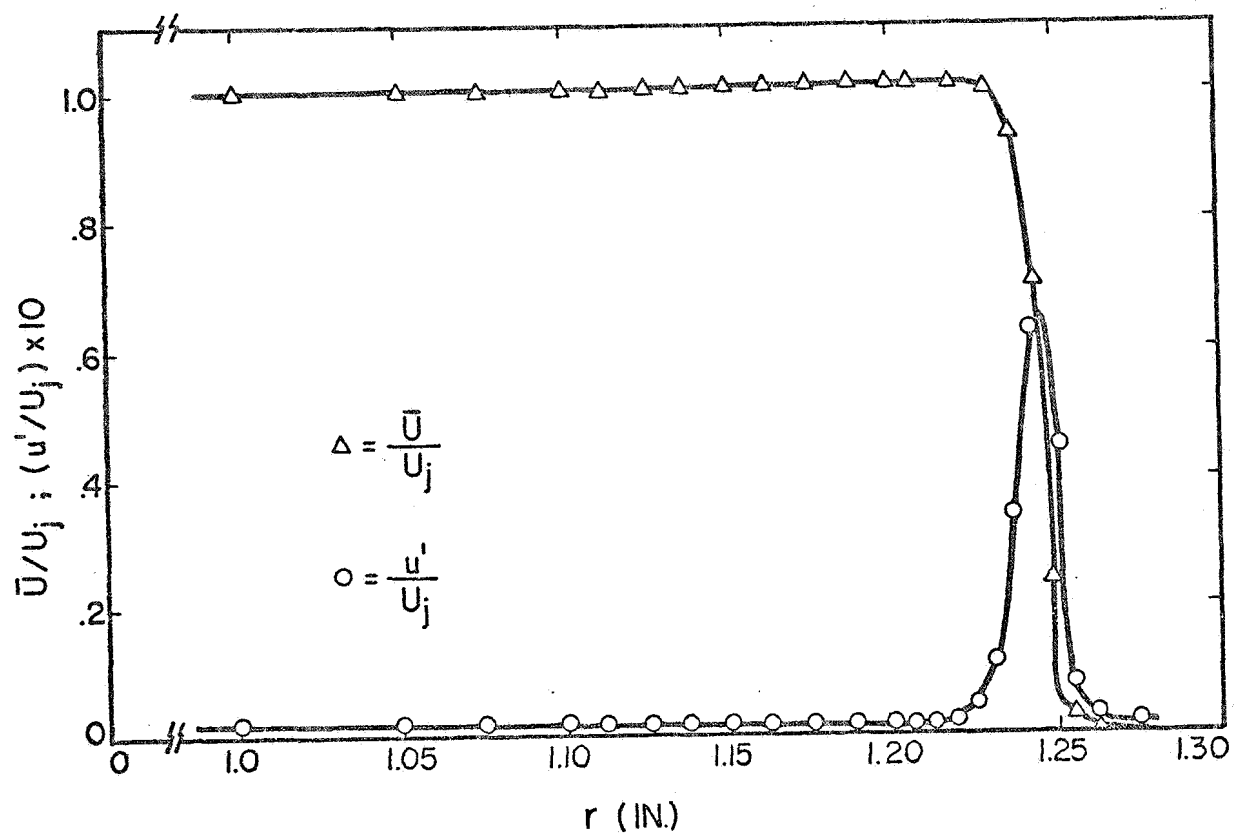


Fig. 5.1-1 Nozzle Exit Conditions: $x = 0.033 \text{ in.}$, $U_j = 300 \text{ ft./sec.}$

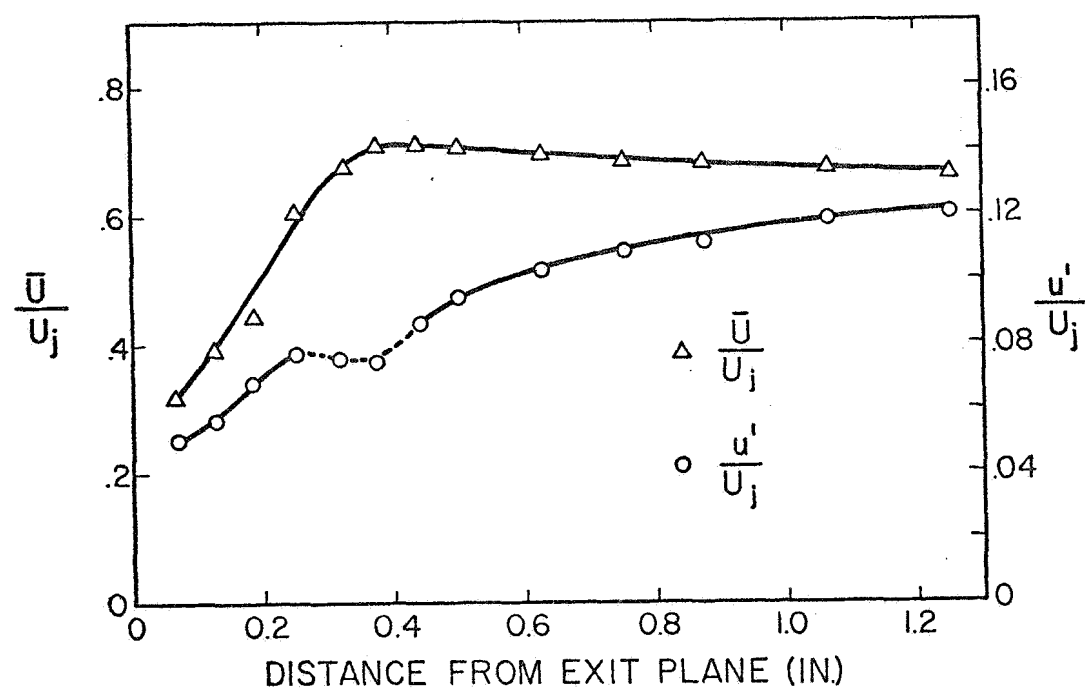


Fig. 5.1-2 Mean and Turbulent Velocity in Developing Region.
 $r/R = 1.0$, $U_j = 300 \text{ ft./sec.}$

measured in or near the jet mixing layer.

Bradshaw (1966) has shown that the exit plane boundary layer considerably affects mixing layer transition and the subsequent approach to similarity. He found that laminar rather than turbulent boundary layers produce the more rapid approach to mixing layer similarity. Additionally, his virtual origin distance varied linearly with the boundary layer Reynolds number.

Hot-wire measurements confirmed a laminar boundary layer .077 in. upstream of the exit plane and an essentially self-similar mixing layer by one diameter downstream. Most of the changes in the approach to similarity take place within 1.0 inch of the exit plane. This region is partially described by axial mean and turbulent velocities directly downstream of the jet lip. Because of the thinness of the initial shear layer, no great accuracy can be claimed for hot-wire measurements in this region, however, the trends should be correct.

The development from the exit plane to 0.25 in. downstream is characterized by slowly increasing turbulent intensities and mean velocity as shown in Fig. 5.1-2. In this initial region, the flow is spectrally characterized by a high-frequency (15kHz) tone imposed upon low frequency (60Hz). Downstream of $x = 0.35$ in., where the turbulent intensity commences a sudden increase, the turbulence spectrum is dominated by the 15 kHz turbulence. Beyond $x = 0.75$ in, the intensities asymptotically approach self-similar values and the spectrum contains increasing amounts of low-frequency turbulence finally merging with the fully developed spectrum. Although a definitive explanation of these observations is not available, it

seems apparent they are closely related to mixing layer transition phenomena. The possibility of probe-Strouhal shedding or electronic noise is eliminated by standard calculations and by the observed lack of disturbance in the smooth jet core flow.

In conclusion, a near ideal jet has been experimentally demonstrated. The exit velocity profile is nearly flat with low disturbance levels. The laminar boundary layer provides a short transition distance to nearly self-similar velocity profiles.

5.2 Mean Velocity

The mean velocity profiles form perhaps the most basic and important descriptions of the jet mixing region. Measurements of the mean velocity were used to verify the symmetry and define the geometry of the jet. At various stages of the experiment, the mean velocity was measured with a small tip impact tube, a single hot wire, or an x probe. Except very near the exit plane where the thin mixing layer makes the single hot wire superior, and in outer high intensity regions where the x probe is more accurate, these three instruments gave nearly the same results. The data presented in this section is from the final measurements made with an x probe.

The mean velocity profiles are shown in a similarity plot in Fig. 5.2-1 and Fig. 5.2-2. The profiles attain a state of self similarity prior to one diameter and maintain it until the end of the potential cone between 4 and 5 diameters. The transition region is characterized by a decreasing centerline velocity as shown in Fig. 5.2-2. In the driven shear layer downstream of the jet lip $\left(\frac{r-R}{x-x_0} = 0\right)$, $\bar{u}/u_j = 0.63$. Alternatively, if the mixing centerline is

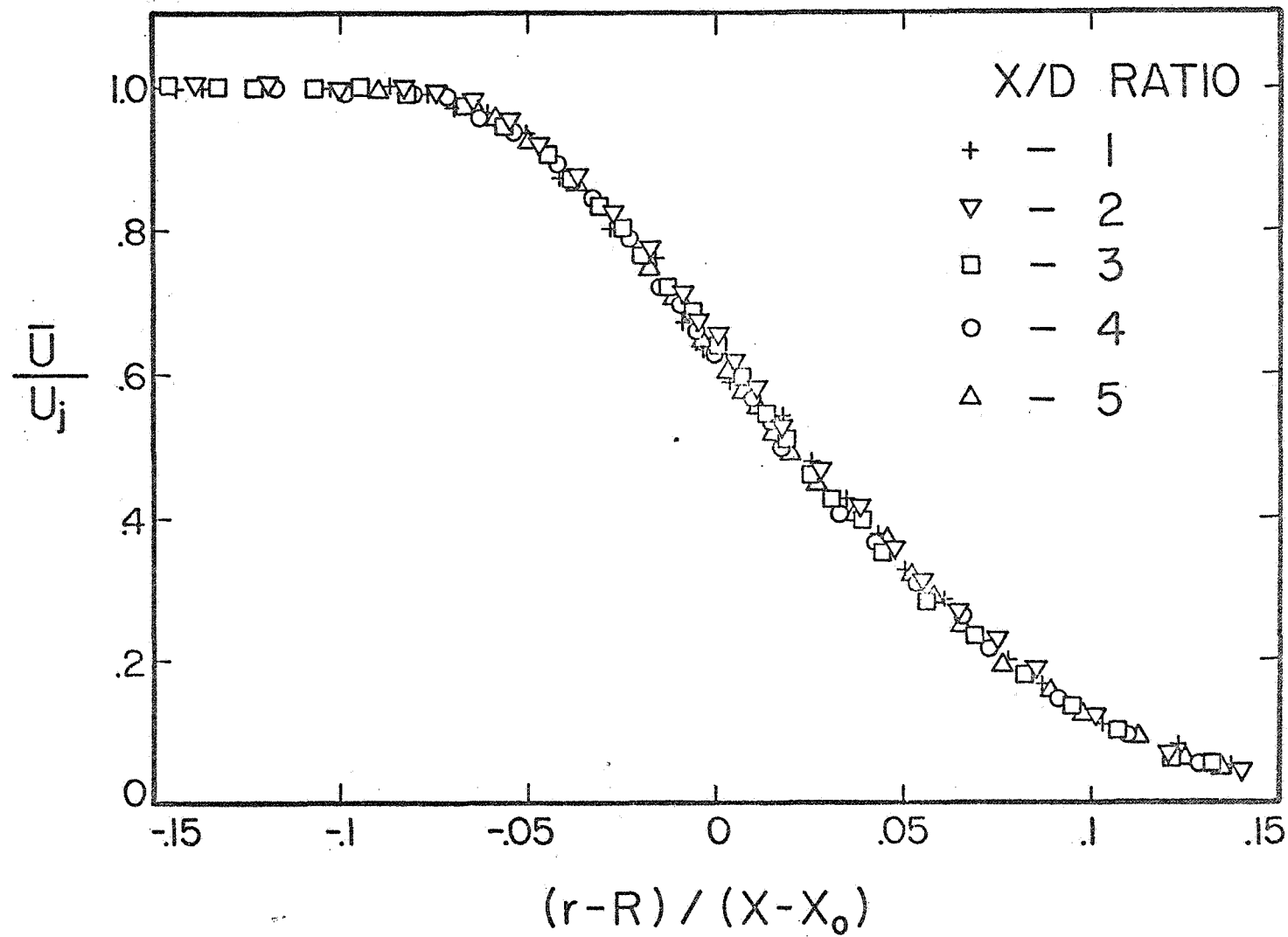


Fig. 5.2-1 Mean Velocity, Driven Mixing Region. $U_j = 300$ ft./sec., $R = 1.25$ in.,
 $x = -0.4$ in.

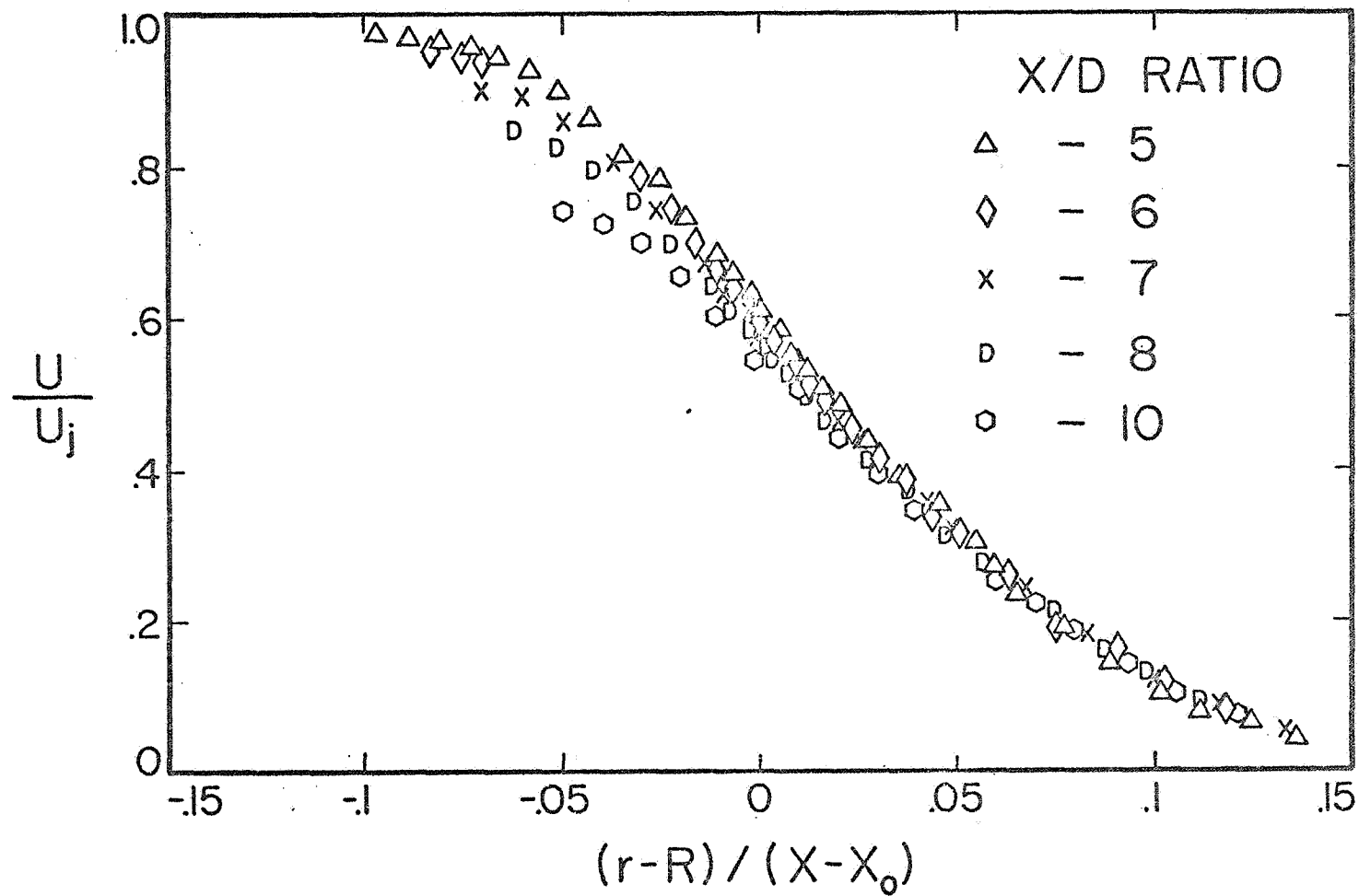


Fig. 5.2-2 Mean Velocity, Transition Region. $U_j = 300$ ft./sec., $R = 1.25$ in.,
 $x = -0.4$ in.

defined in terms of $\bar{u}/u_j = 0.5$, the mixing layer linearly deflects outward on the line $\frac{r-R}{X-X_0} = .018$. Although this outward deflection will be found for other quantities, it is not generally linear. Consequently, it has not been incorporated as a mixing layer similarity parameter.

The virtual origin of the mixing layer was determined from the mean velocity measurements. In the plane mixing layer, this determination is commonly made by extrapolating lines of constant mean velocity toward the beginning of mixing. The intersection fixed X_0 . When attempting to use this method for the jet, it was found that the result varied with the constant mean velocity line selected and the uncertainties were large. This difficulty arises from the nonsymmetric spread of the mixing layer as well as a basic lack of precision in the method. Therefore, an alternate method based on similarity of mixing mean velocity gradient was developed.

Self similarity of the mean velocity gradient requires it to depend only on the similarity position $\eta = \frac{r-R}{X-X_0}$. Thus, if C is a constant,

$$\left| \frac{\partial (u/u_j)}{\partial \eta} \right|_{max} = C \quad (5-1)$$

or

$$\left| \frac{\partial (u/u_j)}{\partial r} (X-X_0) \right|_{max} = C \quad (5-2)$$

Eq. 5-2 is rearranged so that the inverse of the maximum velocity gradient can be plotted for several axial positions in the mixing region

$$\left| \frac{\partial(\bar{u}/u_i)}{\partial r} \right| = \frac{1}{C} x - \frac{1}{C} x_0 \quad (5-3)$$

Equation 5-3 and a standard least squares fitting routine were used to find the virtual origin at $x_0 = -0.4 \pm .1$ in.

The constant C in Eq. 5-3 is also important as a mean shear scale parameter. C was found to be $6.75 \pm .2$. For comparison, this constant may also be related to the Göertler solution spread parameter, σ . The Göertler solution for mean velocity in a plane mixing layer is

$$\frac{\bar{u}}{u_i} = \frac{1}{2} \left[1 - \operatorname{erf} \left(\sigma \frac{r-r_0}{x-x_0} \right) \right] \quad (5-4)$$

Taking the gradient and evaluating at the maximum yields

$$\left. \frac{\partial(\bar{u}/u_i)}{\partial r} \right|_{\max} (x-x_0) = \frac{\sigma}{\sqrt{\pi}} \quad (5-5)$$

Thus,

$$\sigma = \sqrt{\pi} C = 12.0 \pm 0.4$$

These values are in general agreement with those found by other investigators. Davies et al. (1962) found $C = 6$ or $\sigma = 10.7$ with unquantified scatter, in a round jet mixing layer. Liepman and Laufer (1947) found $\sigma = 12$ for a plane mixing layer. Wygnanski and Fiedler (1970) found $\sigma = 9$ for a plane mixing layer. Spencer (1971) determined spread parameters for several velocity ratios in a plane two-stream mixing layer and compared them to Sabin's (1965) analytical result

$$\sigma(\lambda) = \sigma_0 \frac{1+\lambda}{1-\lambda} \quad (5-6)$$

where λ is the ratio of two free stream velocities and σ_0 is the zero velocity ratio parameter, applicable to a simple jet. His comparisons suggested that $\sigma_0 = 11$.

The disagreement in σ values is probably due to two factors--the influence of the flow arrangement upon entrainment and mean velocity measurement errors. The influence of entrainment can be illustrated with Sabin's form in Eq. 5-6. Suppose $\sigma = 12$ was measured in a 300 ft/sec. jet in which the drift velocity outside the jet was 6 ft/sec. Six ft/sec. is a reasonable drift velocity caused by the exhausting jet. The low velocity is almost unnoticeable. The $\sigma(\lambda) = 12$ with Sabin's formula produces a zero velocity ratio $\sigma_0 = 11.5$. Even more extreme perturbations could be expected from non-parallel entrainment flow near a jet exhausting from a large wall.

Mean velocity measurement errors, for both the impact tube and hot wire, stem primarily from the cross component of fluctuating velocity. Spencer's (1970) analysis showed, for the particular impact tube used, that the errors near the mixing centerline would be less than 1%. Davies et al. (1963) found the same order of error for a single hot wire near the center of mixing. Spencer's response equations, however, show an x probe response contains an inherent correction. We, therefore, conclude that the fluctuation error in mean velocity measurements is less than 1%.

The mean velocity gradient was shown in Chapter 2 to be a very important constituent of the pressure intensity source. The gradient was determined from the velocity profiles by using both graphical techniques and an IBM Scientific Subroutine DGT-3. Agreement was good. The axial distribution, directly downstream of the jet lip, is shown in Fig. 5.2-3. The inverse relation of gradient and axial position is seen to be good in the driven region ($x/D < 5$) and reasonable in the transition region ($5 < x/D < 10$). The small deviations from constant $\frac{\partial(\bar{u}/u_i)}{\partial r}(x-x_0)$ are due to a combination of development of the mixing region for small x/D , and outward deflection of the jet at larger axial distances. In the transition region, the scaling could only be expected to be approximate. The transverse distribution of gradient is shown in Fig. 5.2-4. An important feature is the slowly changing value in a region about the center of mixing.

5.3 Point Turbulent Velocity Structure

5.3.1 Point Turbulent Velocity Second Order Correlations

As was demonstrated in Chapter 2, the second order velocity correlations were necessary to describe the pressure field. It is convenient to consider these correlations as elements of the second order velocity tensor $\overline{u_i u_j}$. The diagonal elements $\overline{uu}$, $\overline{vv}$, and $\overline{ww}$ are proportional to turbulent kinetic energy. Alternately their square root, directly related to the intensity, is a measure of the size of turbulent fluctuations. The $\overline{uv}$ element is commonly known to be proportional to the turbulent shearing stress.

The velocity tensor may be expressed equivalently in terms of the axial mean square velocity and structural constants. The constants are defined as:

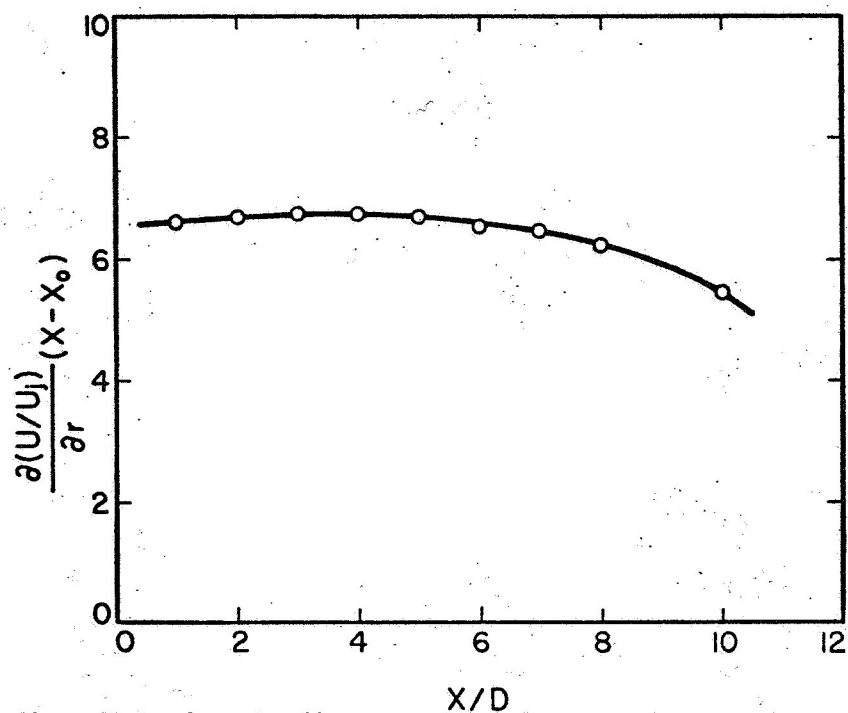


Fig. 5.2-3 Mean Velocity Gradient, Axial Distribution. $r/R = 1.0$.

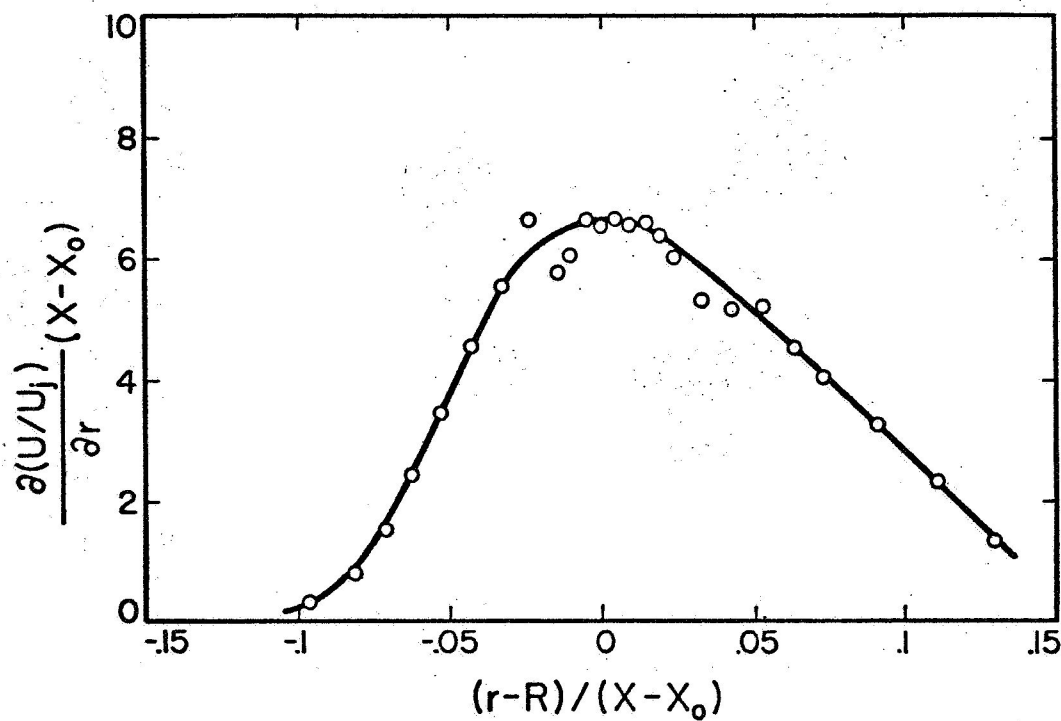


Fig. 5.2-4 Mean Velocity Gradient, Transverse Distribution. $x/D = 4.0$,
 $U = 300$ ft./sec., $R = 1.25$ in., $x = -0.4$ in.

$$k = \left(\frac{\overline{v v}}{\overline{u u}} \right)^{1/2} \quad \text{Intensity Anisotropy Ratio} \quad (5-7)$$

$$k_z = \left(\frac{\overline{w w}}{\overline{u u}} \right)^{1/2} \quad \text{Intensity Anisotropy Ratio} \quad (5-8)$$

$$R_{uv} = \frac{\overline{uv}}{(\overline{u^2} \overline{v^2})^{1/2}} \quad \text{Shear Correlation Coefficient} \quad (5-9)$$

The tensor may then be expressed in terms of its intensity and structural parts

$$\overline{u_i u_j} = \overline{U^2} \begin{vmatrix} 1 & k R_{uv} & 0 \\ k R_{uv} & k^2 & 0 \\ 0 & 0 & k_z^2 \end{vmatrix} \quad (5-10)$$

in which there is no mean gradient related to the w -component. The angle of principal stress is another important structural parameter. In a coordinate system rotated to the angle β , all shearing stresses vanish. In terms of the structural constants, β is given by

$$\beta = \frac{1}{2} \tan^{-1} \left(\frac{2 R k}{1 - k^2} \right) \quad (5-11)$$

The turbulent axial, transverse, and lateral turbulence intensities for the driven shear region are shown in Figs. 5.3-1, 5.3-2 and 5.3-3. These results are in general agreement in magnitude and trends

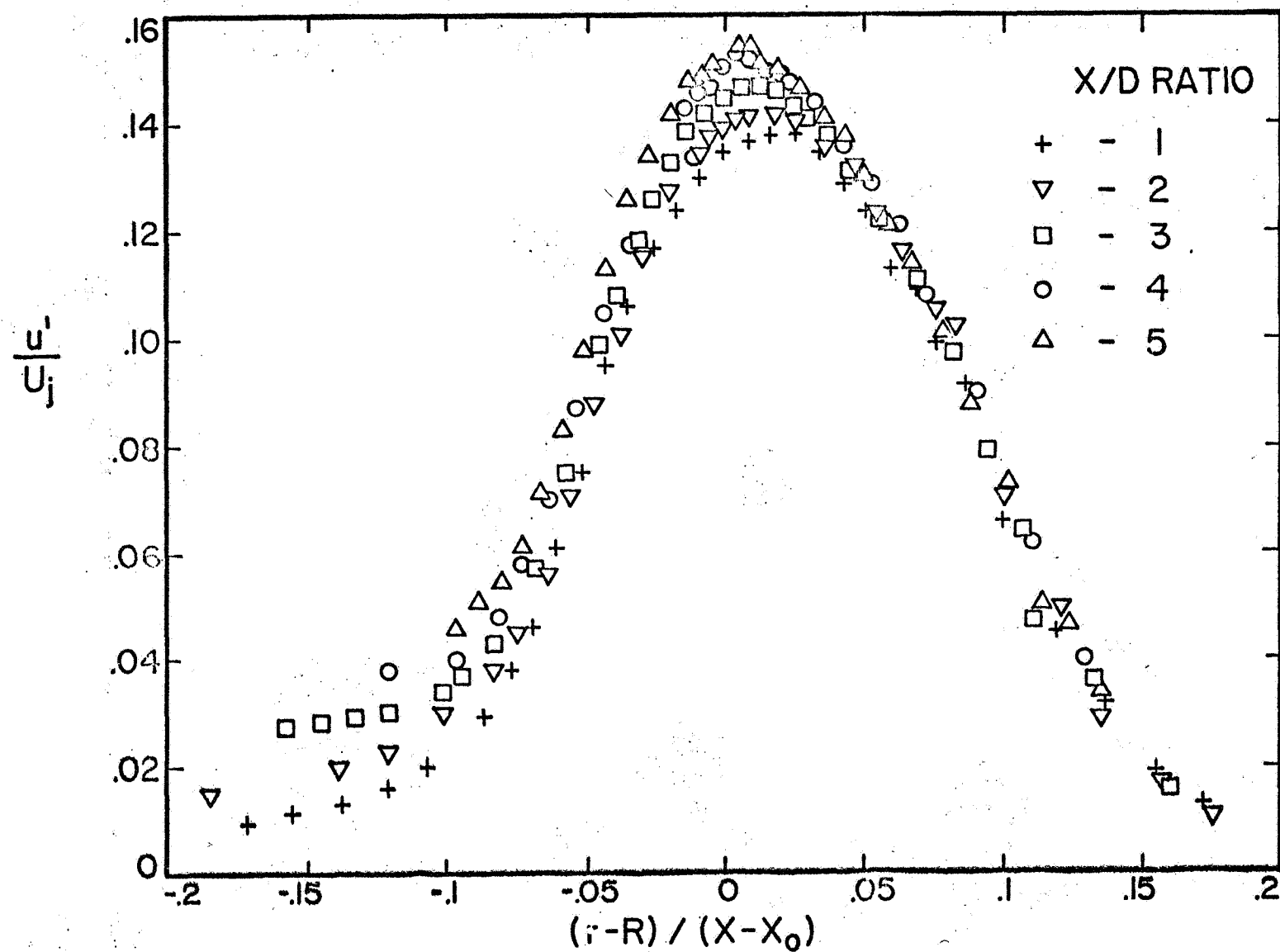


Fig. 5.3-1 Axial Turbulent Velocity Fluctuations, Driven Mixing Region. $U = 300$ ft./sec.,
 $R = 1.25$ in., $x = -0.4$ in.

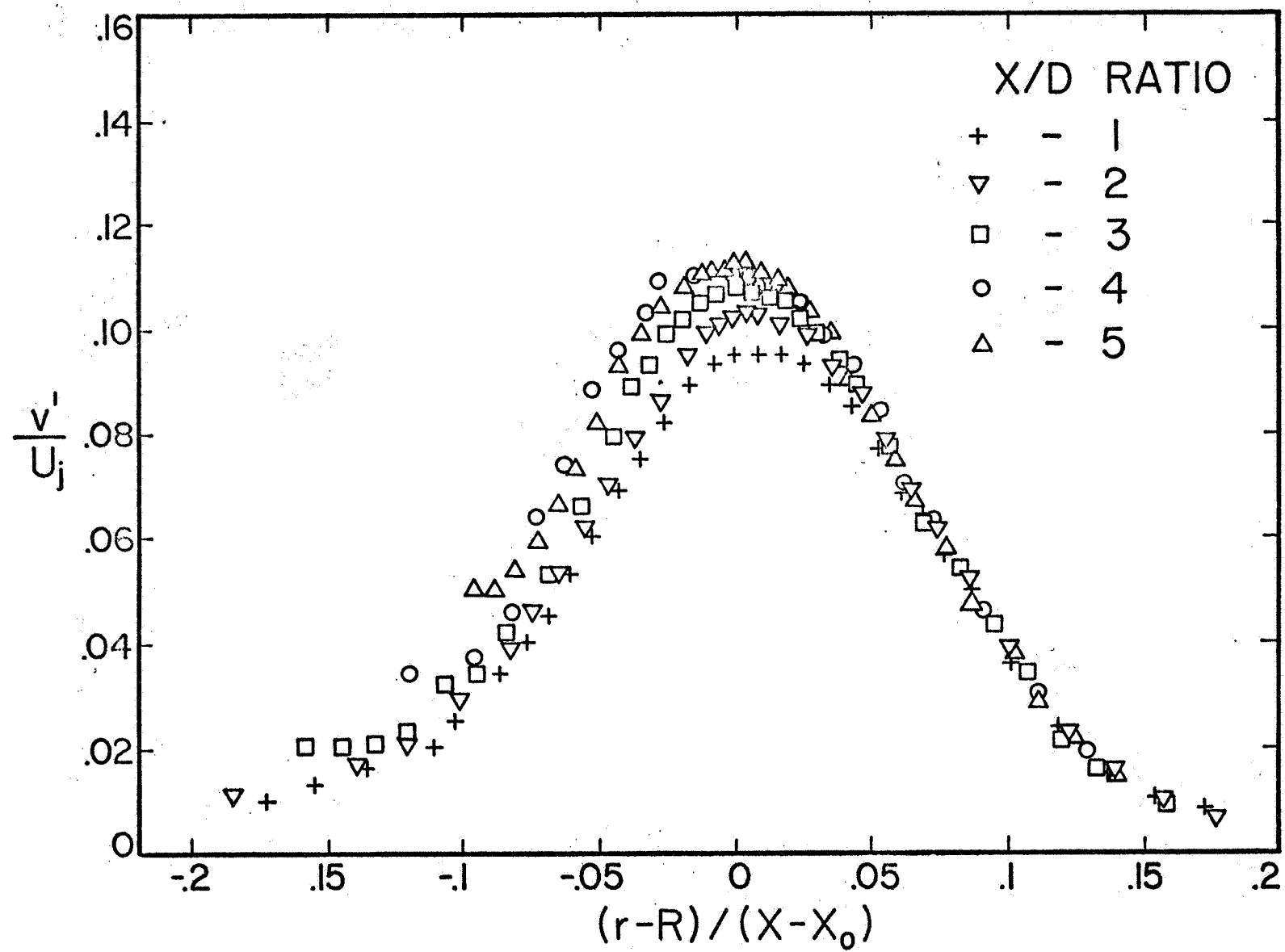


Fig. 5.3-2 Transverse Velocity Fluctuations, Driven Mixing Region. $U_j = 300$ ft./sec.,
 $R = 1.25$ in., $x = -0.4$ in.

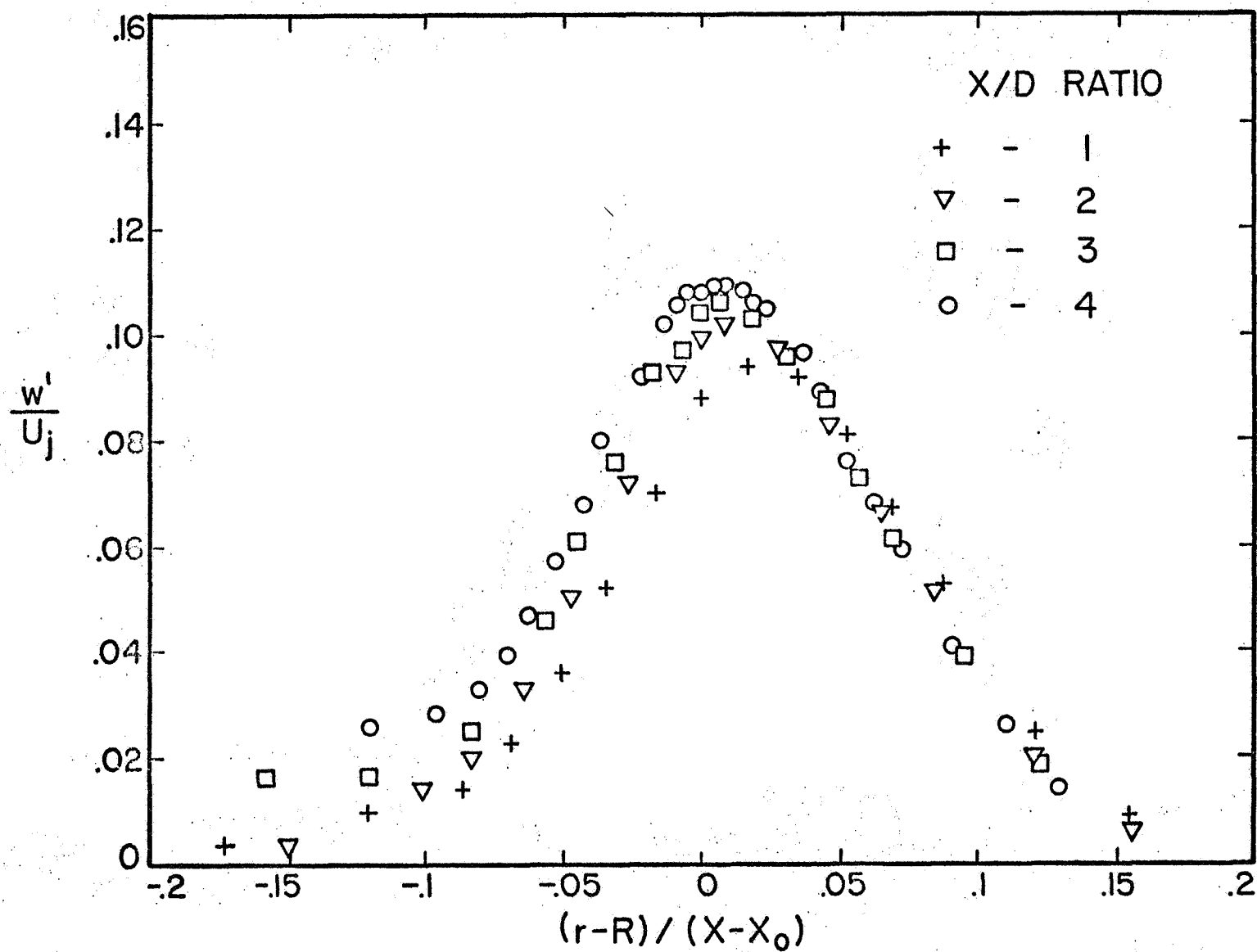


Fig. 5.3-3 Lateral Velocity Fluctuations, Driven Mixing Region. $U_j = 300$ ft./sec., $R = 1.25$ in., $x_0 = -0.4$ in.

with the classic round jet literature. The associated results for the transition region beyond $x/D = 5$ are shown in Figs. 5.3-4, 5.3-5 and 5.3-6.

The intensities are observed to reach only approximate similarity. All three components gradually increase with axial position throughout the driven region. A broad maximum occurs between 4 and 6 diameters before the expected decrease in the transition region. In contrast with the mean velocity profiles, which deflected outward, the peak intensities occur near $\frac{r-R}{X-X_0} = .02$ at $x/D = 1$ and gradually deflect toward $\frac{r-R}{X-X_0} = 0$ as axial position increases.

The shear stress $\overline{uv}$ is shown in Fig. 5.3-7 for the driven region and in Fig. 5.3-8 for the transition region. The departures from similarity are generally the same as observed for the turbulence intensities.

Because of the lack of complete similarity in both the driven and transition regions, we press the search for flow invariance one step further. It is meaningful physically, and convenient for subsequent analysis, to use the structural-intensity division as previously discussed. The point turbulence is then described in terms of an intensity and structural parameters. Figure 5.3-9 shows that the important structural parameters k and k_{uv} change very little with axial position not only in the driven but in the transition region as well. As a parallel observation, the driven region velocity intensity profiles, when normalized by peak intensity and compensated for deflection, have similar shapes. The important conclusion is that while turbulence intensities vary slowly with axial position, the turbulent structure

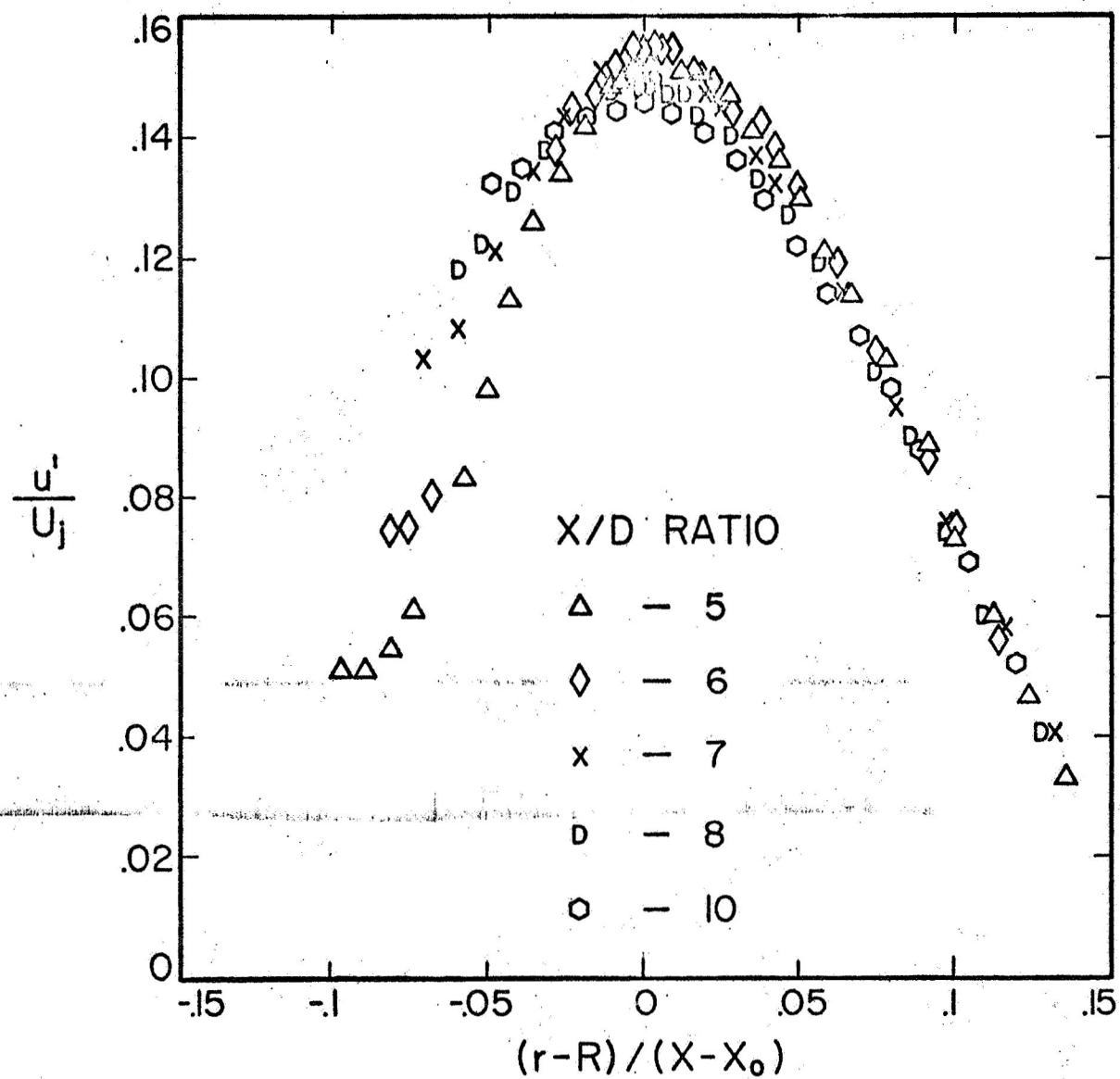


Fig. 5.3-4 Axial Velocity Fluctuations, Transition Region. $U = 300$ ft./sec., $R = 1.25$ in.,
 $x = -0.4$ in.

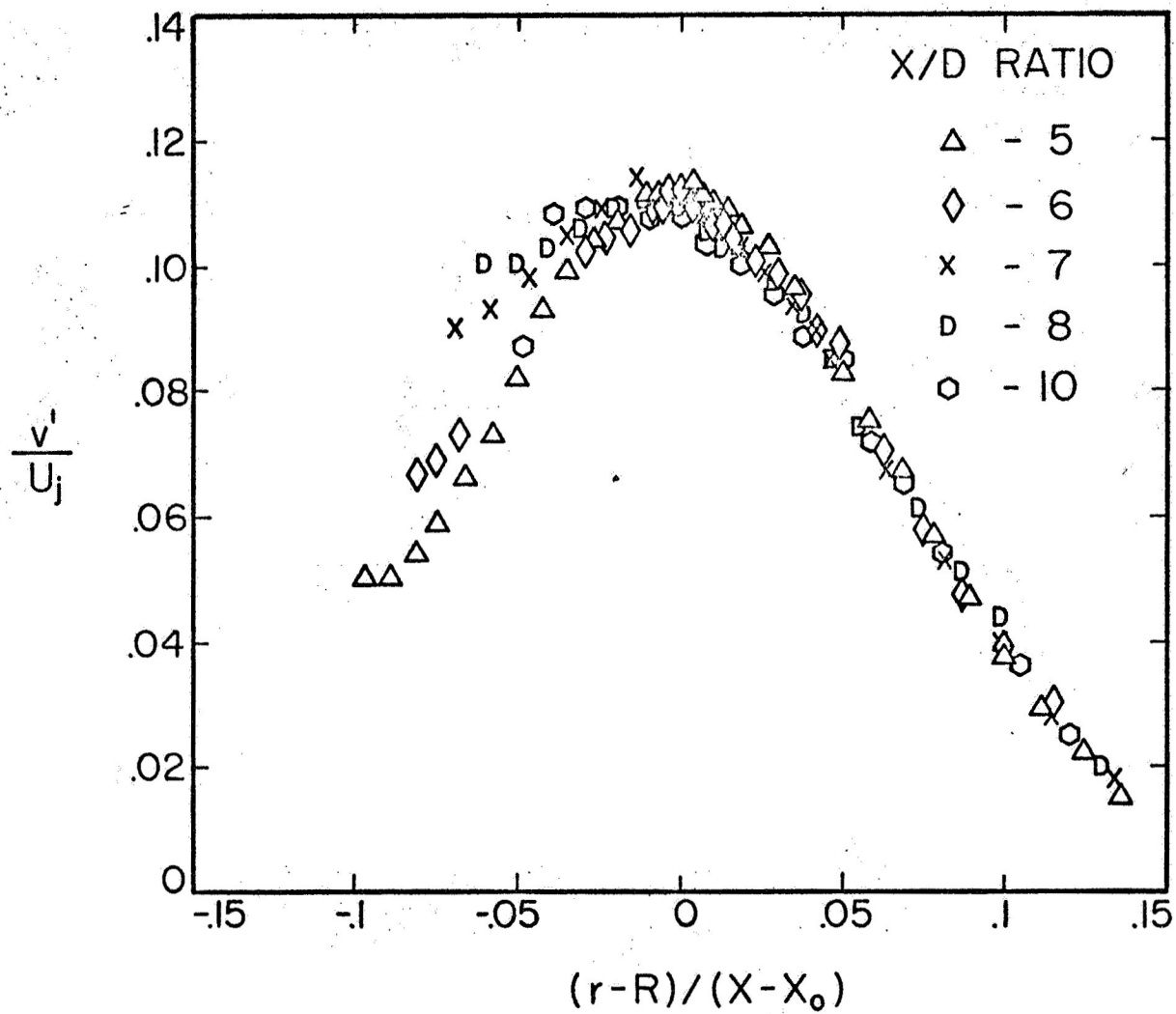


Fig. 5.3-5 Transverse Velocity Fluctuations, Transition Region. $U_j = 300$ ft./sec.,
 $R = 1.25$ in., $x = -0.4$ in.

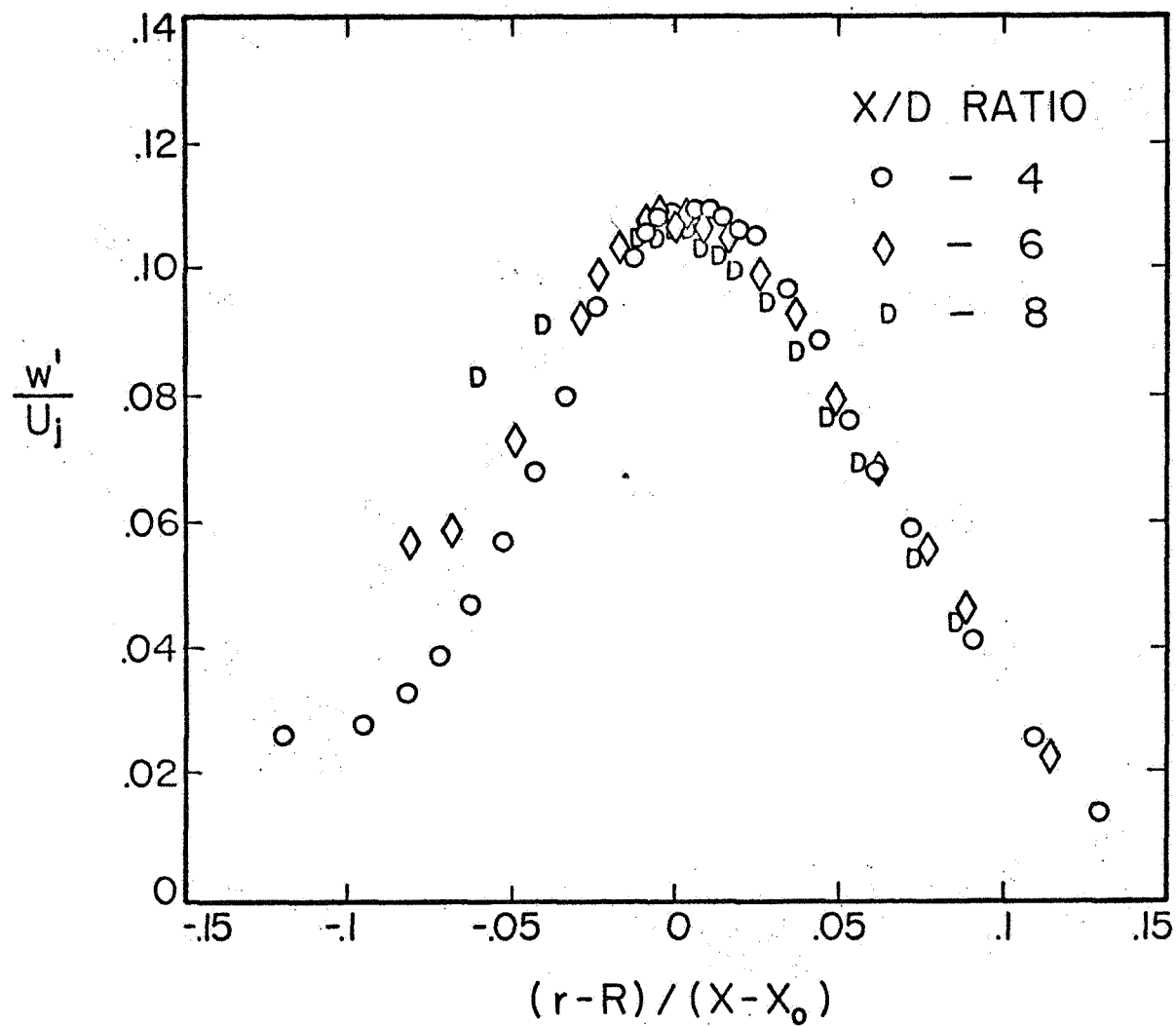


Fig. 5.3-6 Lateral Velocity Fluctuations, Transition Region. $U_j = 300$ ft./sec., $R = 1.25$ in., $x_j = -0.4$ in.

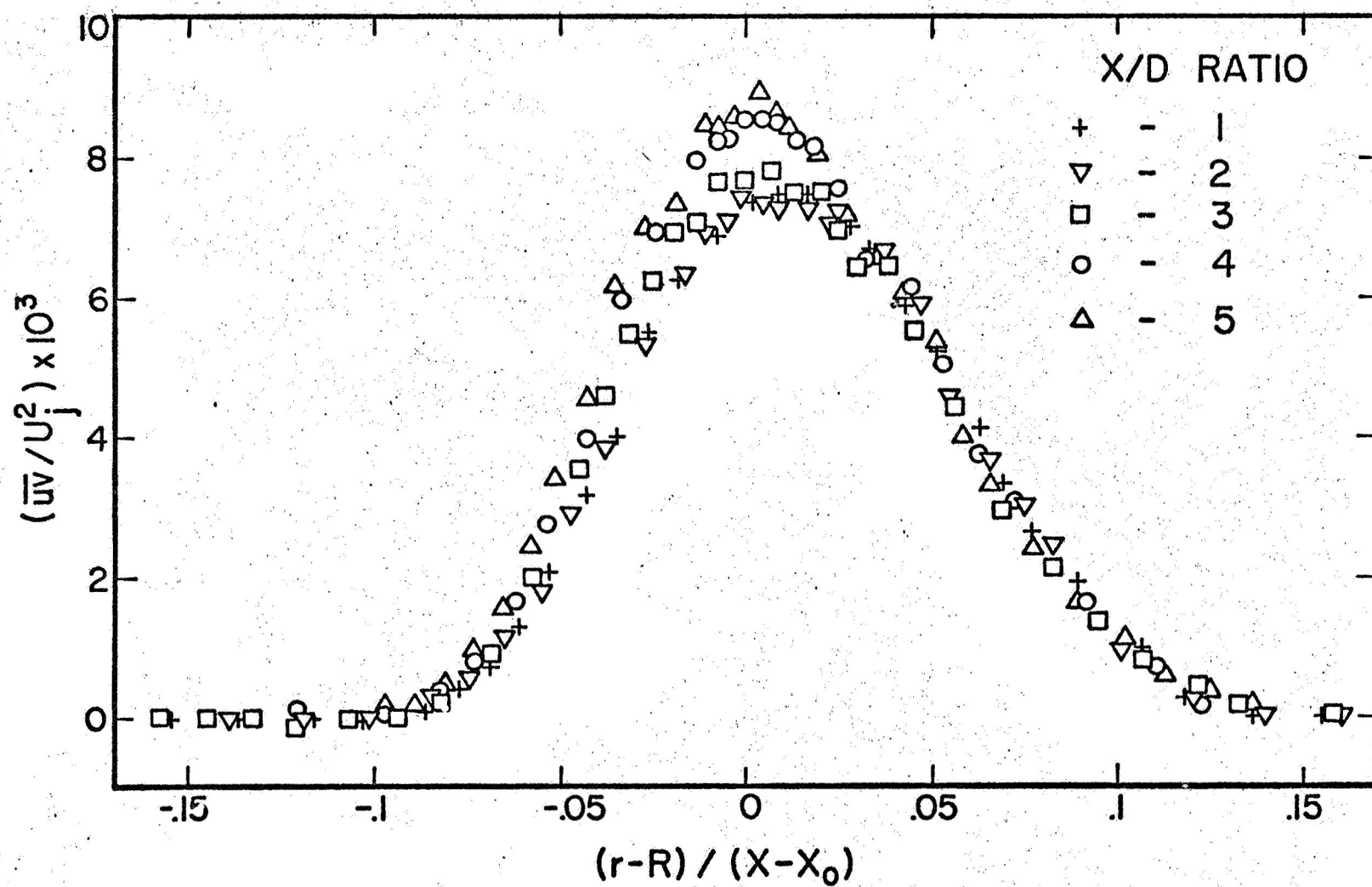


Fig. 5.3-7 Reynolds Shear Stress Driven Mixing Region. $U_j = 300$ ft./sec., $R = 1.25$

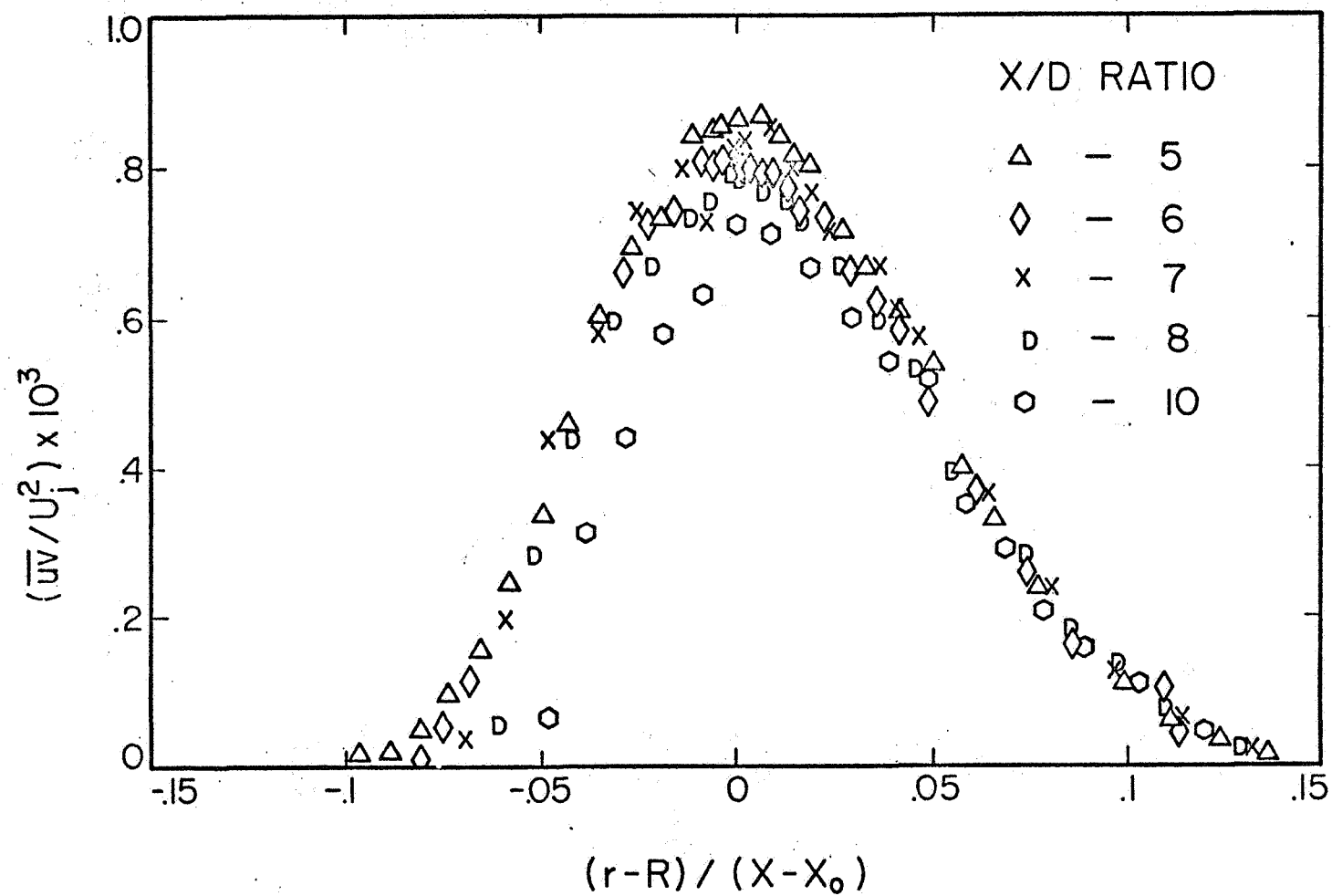


Fig. 5.3-8 Reynolds Shear Stress, Transition Region. $U_j = 300$ ft./sec., $R = 1.25$ in., $x_0 = -0.4$ in.

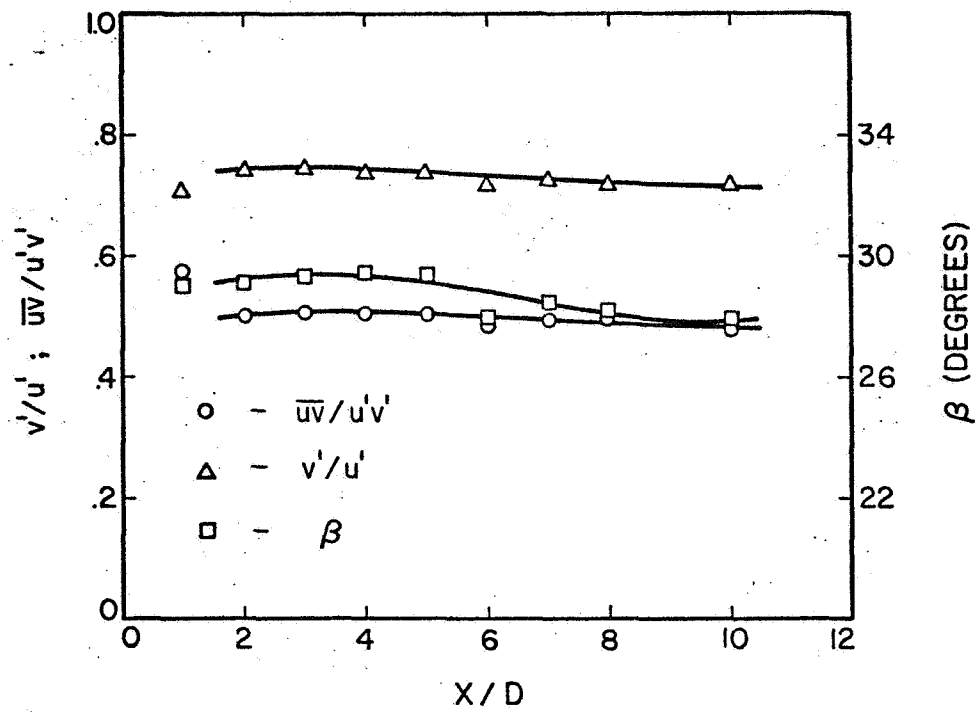


Fig. 5.3-9 Turbulent Structure Constants, $r/R = 1.0$.

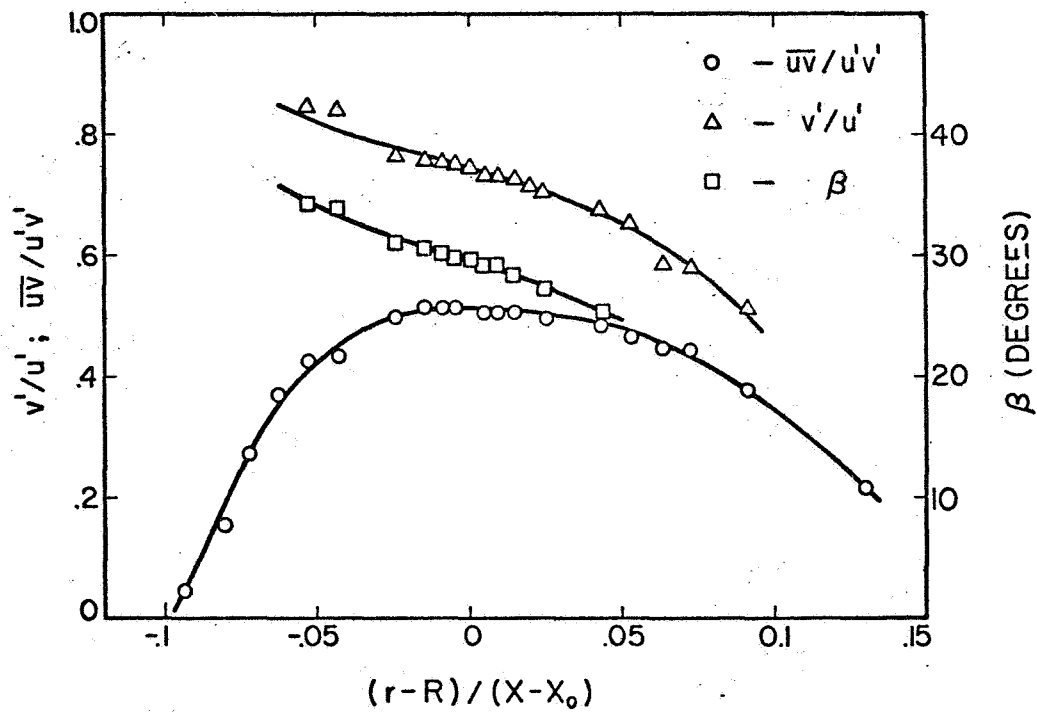


Fig. 5.3-10 Turbulent Structure Constants, $x/D = 4$, $U = 300$ ft./sec., $R = 1.25$ in., $x = -0.4$ in.

is practically invariant in the driven region and well into the transition region. Although not as dramatic as the axial invariance, the transverse variation of structure is very slow. Figure 5.3-10 shows the slowly varying structural constants at $x/D = 4$.

In summary, experimental results for second order velocity correlations were presented. The results, when treated as elements of a second order tensor, may be divided into intensity and structural parts. The structural part was found to be self similar and to vary slowly with transverse position. The intensity component, in contrast, never becomes fully self similar and has sizable transverse variation.

5.3.2 Point Turbulent Velocity Higher Order Correlations

As was shown in the theory for the pressure transducer, a knowledge of the turbulent structure is necessary to predict the transducer errors. The purpose in presenting the skewness, flatness, and other higher order moments, therefore, is to present the experimental input necessary for the PT error theory. The data, however, is of more general importance for it further describes the turbulent statistical structure and provides insight to terms describing turbulent transport.

The skewness and flatness of the three components of velocity are shown in Figs. 5.3-11 and 5.3-12. The skewness and flatness are observed to vary similarly while the w component seems more Gaussian away from the mixing centerline. The trends of nearly Gaussian behavior near the centerline were observed by Spencer (1970) and others. Weber's (1974) results show similarity at various axial

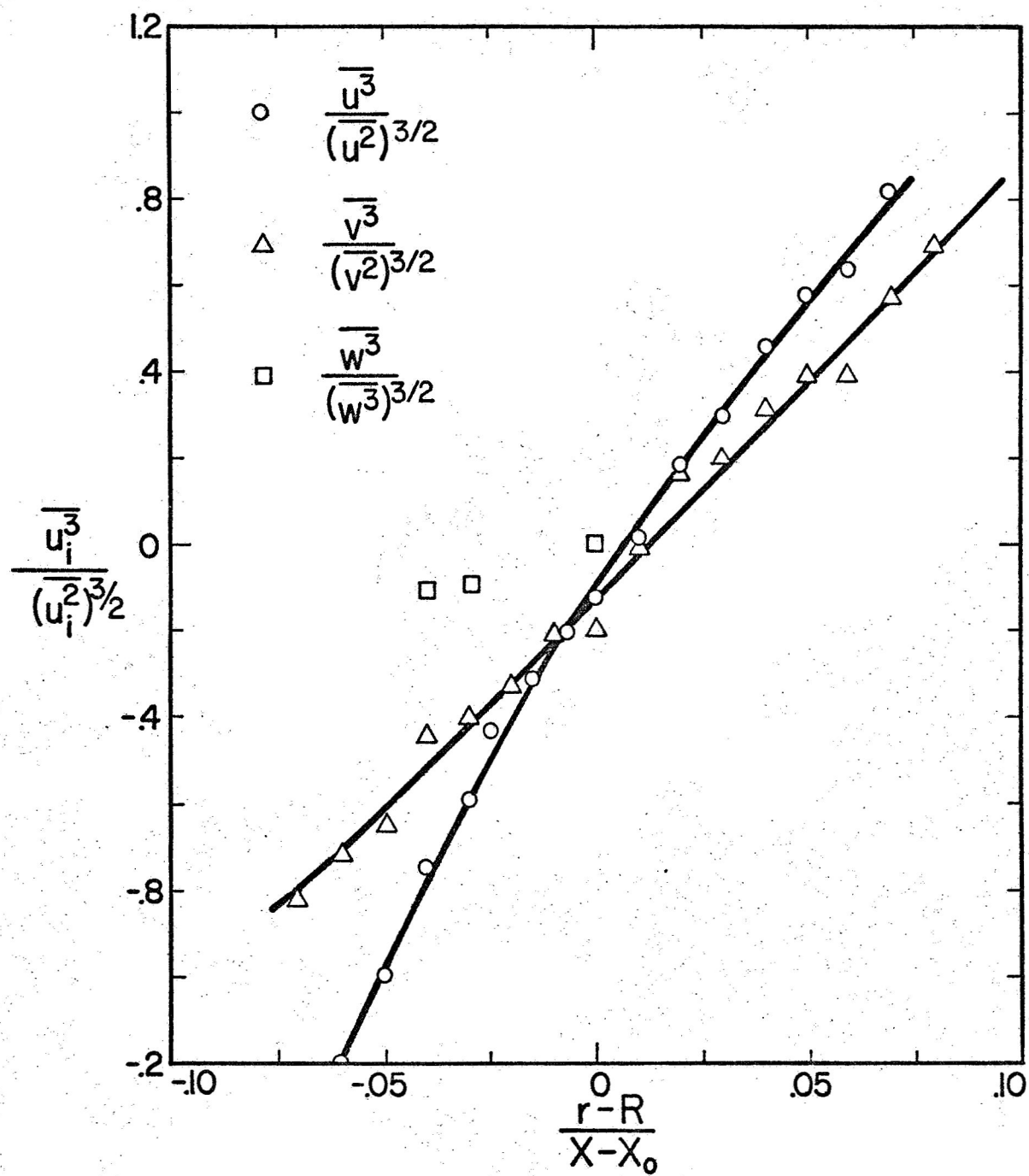


Fig. 5.3-11 Velocity Skewness, $x/D = 4$, $U = 300$ ft./sec., $R = 1.25$ in.,
 $x_o = -0.4$ in.

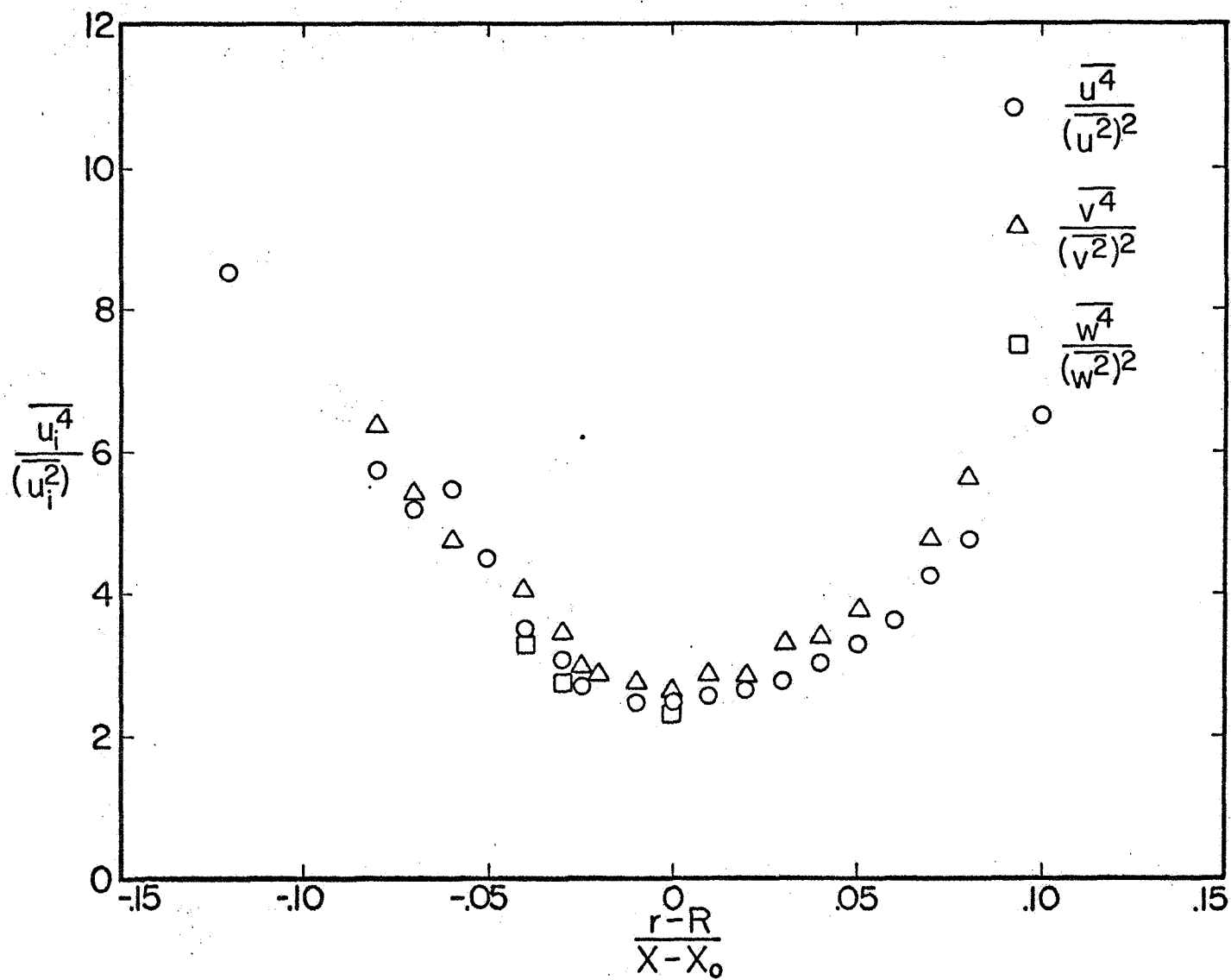


Fig. 5.3-12 Velocity Flatness. $X/D = 4$, $U_j = 300$ ft./sec, $R = 1.25$ in., $x_o = -0.4$ in.

positions. When comparing these results with others, it should be remembered that the transverse coordinate r is defined so that $\partial \bar{u} / \partial r < 0$. If the opposite were true, structural constants continuing odd powers of V would change sign. Results of the fourth order correlations are shown in Fig. 5.3-13. Like skewness and flatness both $\overline{u^2 v^2} / \bar{u}^2 \bar{v}^2$ and $\overline{u^2 w^2} / \bar{u}^2 \bar{w}^2$ are nearly normal near the mixing centerline and depart rapidly from this condition toward the outer edges. The third order constants are shown in Fig. 5.3-14. Townsend (1956) interpreted these correlations as turbulent transport coefficients and deduced the velocity of turbulent energy convection. For example, $\overline{u^2 v} / \bar{u}^2 \bar{v}$ is the coefficient describing convection of $\bar{u}^2$ kinetic energy by the V component of velocity. Further, the convection velocity for this transport is given by $v^* = \overline{u^2 v} / \bar{u}^2$. With this interpretation, the trends in Fig. 5.3-14 have real physical significance and agree with those observed by Spencer. Specifically, v^* convects axial kinetic energy away from the mixing centerline. u^* convects both transverse and lateral kinetic energy downstream on the low velocity side and upstream on the high velocity side. The lateral transport term $\overline{w u^2} / \bar{w} \bar{u}^2$ was found to be zero as it should be in a symmetric jet.

In summary, the higher order correlations were measured for input to the pressure transducer error model. The results agree with the findings of others and classic physical interpretations.

5.3.3 One-Dimensional Spectra

The one-dimensional spectrum, as presented by Hinze (1959) can be used to describe the distribution of kinetic energy of a fluctuating velocity component or, with approximation, the distribution of

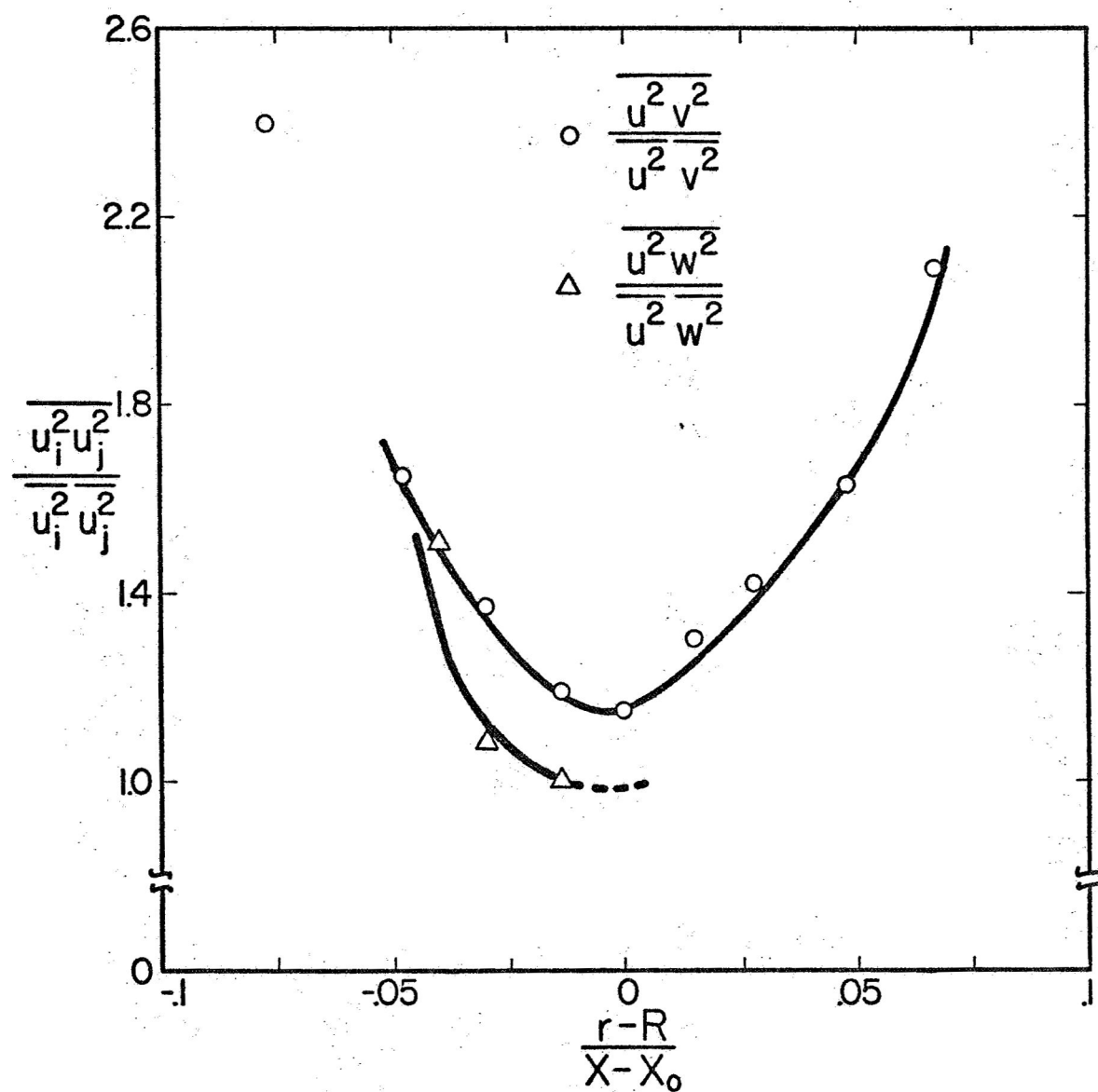


Fig. 5.3-13 Fourth Order Velocity Correlations. $x/D = 4$, $U_j = 300$ ft./sec.,
 $R = 1.25$ in., $x = -0.4$ in.

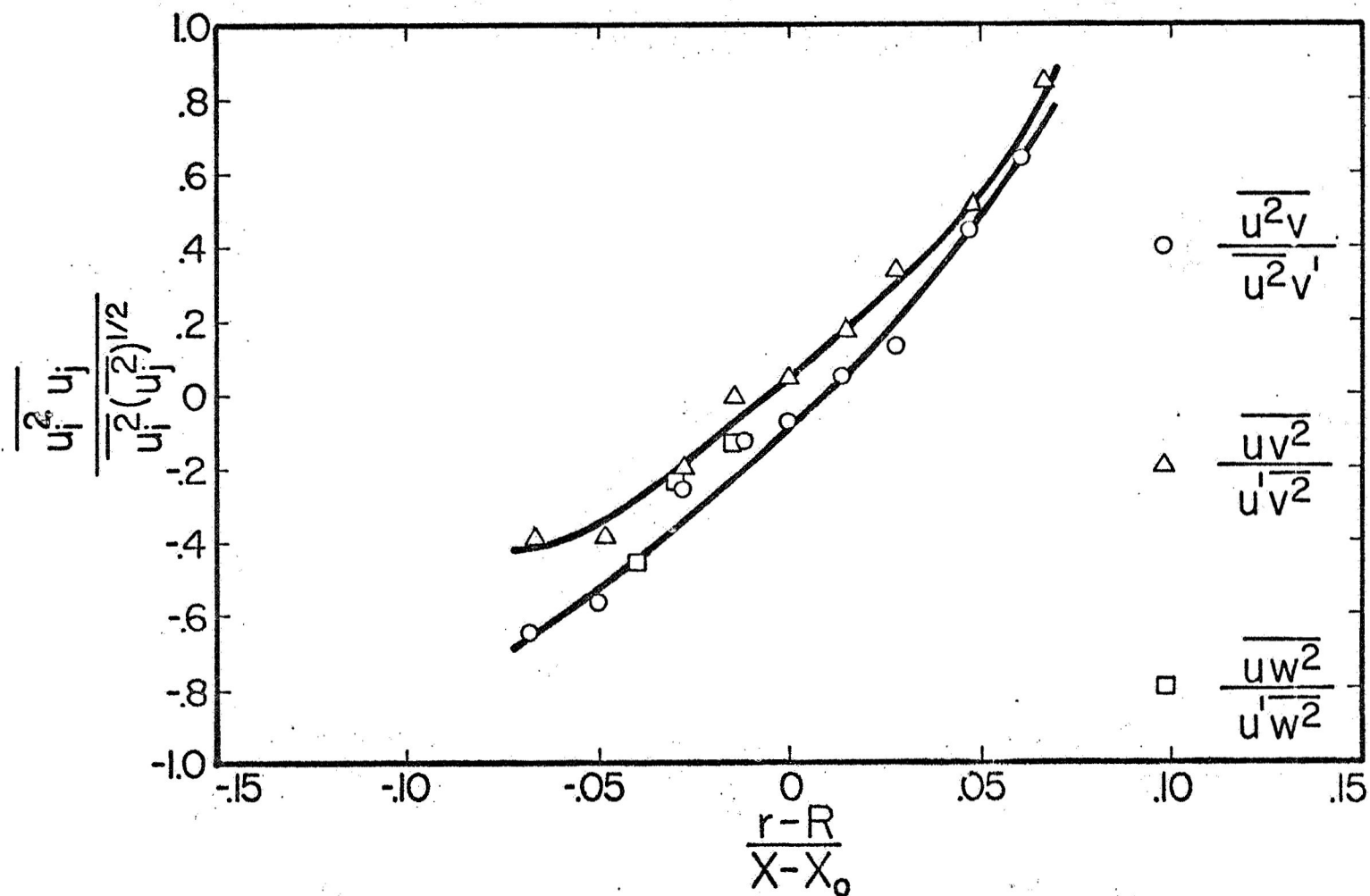


Fig. 5.3-14 Third Order Velocity Correlations. $x/D = 4$, $U_j = 300$ ft./sec., $R = 1.25$ in.,
 $x = -0.4$ in.

convected eddy sizes. We define $F_u(f)$ as the fraction of energy per unit cycle at frequency f in a fluctuating signal u . As Hinze points out, the energy spectrum of a velocity component can be determined from its auto-correlation function, or as was done in our case, by filtering the analog signal of the velocity. The spectra were generated by a 1/3 octave band pass filter. For unity input, the output mean square voltage for each center frequency was then the fraction of kinetic energy in the 1/3 octave band. The result was put on a unit cycle basis by dividing by band width.

The spectral distributions are presented in the common nondimensional form based on Taylor's frozen pattern hypothesis. If the pattern of turbulence is frozen, the observed time variation of a fixed point is due to spatial variations being convected past. Therefore, the nondimensional coordinates $\frac{F(f)u}{L}$ vs. $\frac{fL}{u}$ should collapse structurally similar spectral distributions providing u and L are velocities and lengths proportional to the local convection speed and scale size.

Figure 5.3-15 and Fig. 5.3-16 show spectral distribution of the axial and transverse velocity components on the mixing centerline at several downstream positions. These distributions, in agreement with the other results, suggest a nearly frozen pattern, structural similarity, constancy of the convection velocity, and linearly-growing scales. The $-5/3$ power decrease indicates the Kolmogoroff inertial subrange discussed by Hinze (1959). The pronounced peak of $F_v(f)$ in the mixing region corresponds to a spectral peak of $F_u(f)$ on the jet centerline and has been observed by several others. Lau,

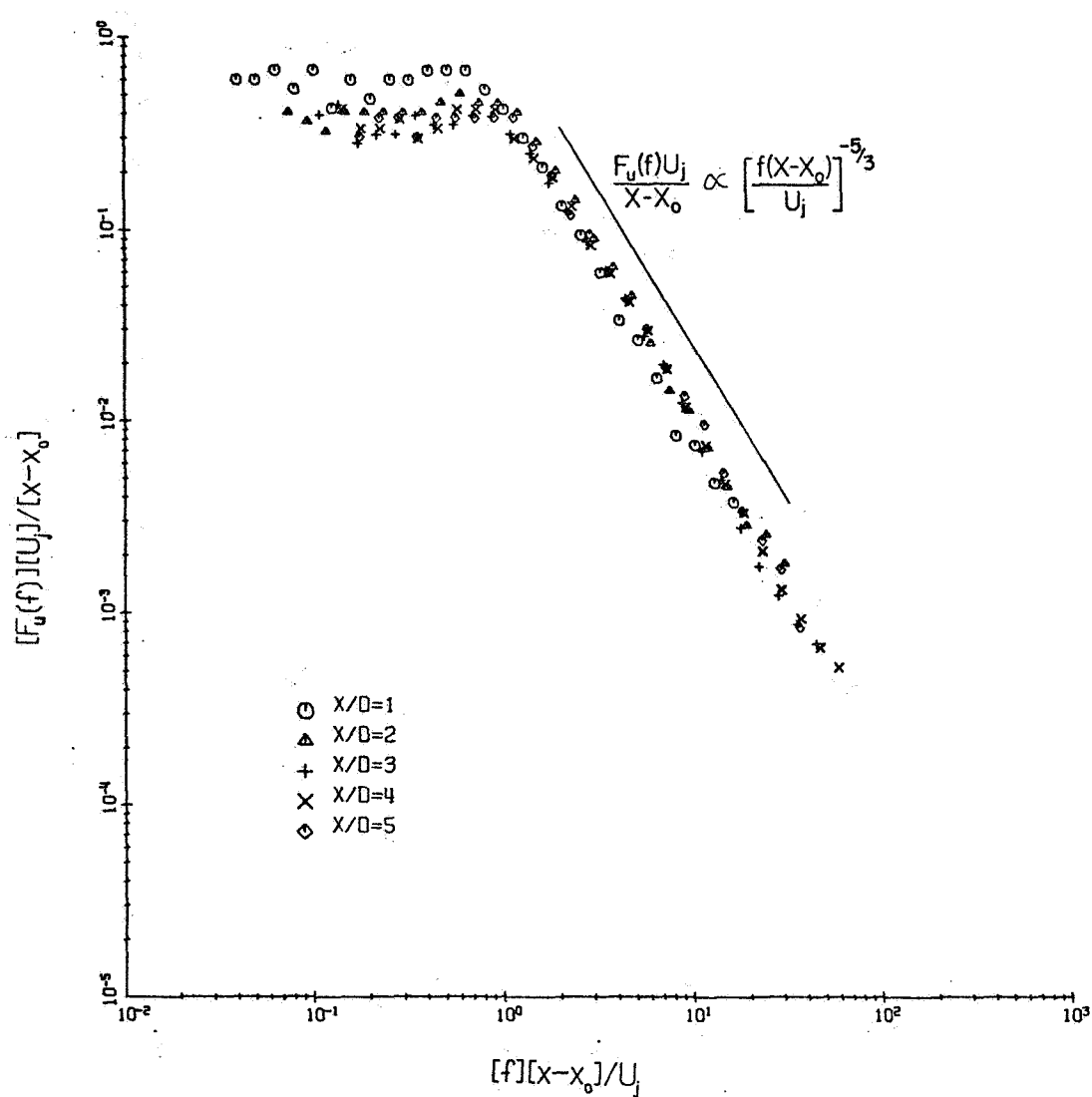


Fig. 5.3-15 Spectra of Axial Fluctuating Velocity. $r/R = 1.0$,
 $U = 300$ ft./sec., $x = -0.4$ in.

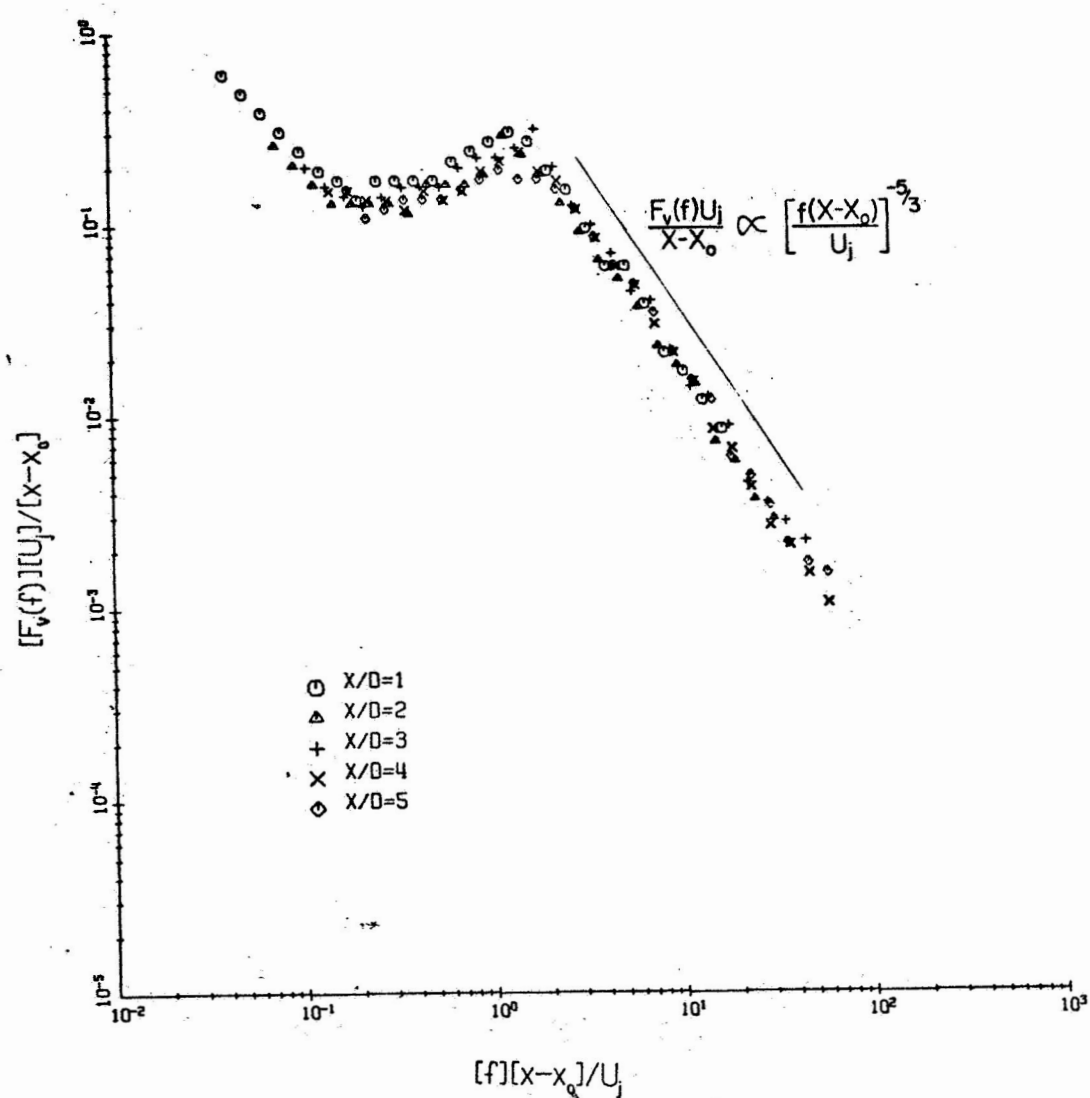


Fig. 5.3-16 Spectra of Transverse Fluctuating Velocity. $r/R = 1.0$,
 $U = 300$ ft./sec., $x = -0.4$ in.

Fisher, and Fuchs (1972) proposed a mixing model featuring large convected vortices that is consistent with this observation. Jones, Hammersley, Planchon, and Spencer (1973) related the spectral peak to oscillating space correlations and suggested a large scale structure as an explanation. Also, as we will see in Chapter 6, the spectral peak for pressure $F_p(f)$ compares almost directly to the peak in $F_v(f)$.

5.4 Two-Point Velocity Field Structure

Extension beyond the point turbulent field results discussed above, leads to the next more detailed level of description; the velocity space-time correlations. Attention is restricted to the two-point correlation of a single velocity at each point. The physical significance of this space-time correlation was stated by Favre (1965): "The space-time correlation gives evidence of the heredity of turbulence and defines scales indicative of the time and spatial variation of the turbulence structure." A specific application was identified in Chapter 2. There, the velocity space-time correlation was shown to be an integral-forcing function for the pressure space-time correlation.

The space-time correlations are defined by

$$R_{u_i u_j}(x, r, z, t, \Delta x, \Delta r, \Delta z, \tau) = \frac{\overline{u_i(x, r, z, t) u_j(x + \Delta x, r + \Delta r, z + \Delta z, t + \tau)}}{\left(\overline{u_i^2(x, r, z, t)} \overline{u_j^2(x + \Delta x, r + \Delta r, z + \Delta z, t + \tau)} \right)^{1/2}} \quad (5-12)$$

The position coordinates x, r, z refer to the position of a fixed point and $\Delta x, \Delta r, \Delta z$ are the relative separation distances of the second

point. In like fashion, $\mathcal{T}$ is a delay time. Stationary turbulence was observed so the correlation is not a function of $\mathcal{T}$. For brevity, the position arguments and separation arguments which are equal to zero are suppressed.

As previously discussed, two-point measurements were made at several positions along the mixing centerline and at several transverse positions at $x/D = 4$. Data was gathered primarily for separations along the jet coordinate axis. Weber (1974) reports the full set of results. Here we have primarily concentrated on correlations most directly related to the pressure field: namely, R_{uu} and R_{vv} .

Examples of $R_{uu}(\Delta x, \mathcal{T})$ and $R_{vv}(\Delta x, \mathcal{T})$ are in Fig 5.4-1 and 5.4-2. Each curve in these families is a correlation between a fixed position x probe and another x probe separated Δx . Families of curves for other fixed positions differ only in time and space scale. A lateral x probe offset prevented the correlations from approaching 1.0 for small radial and axial separations. This offset was necessary to avoid upstream probe interference as explained in Chapter 4.

Favre's (1965) statement about space-time correlations provides the outline for the next two subsections. In the first subsection, the concept of the convected frame will be introduced and experimental evidence describing its convection speed and time evolution will be presented. In the second subsection, pertinent features of the spatial structure of the turbulent velocity field will be described.

5.4.1 Convected Frame Turbulence

The sense in which we consider a convected frame was defined by Wills (1964) as that frame in which the turbulent time scale is

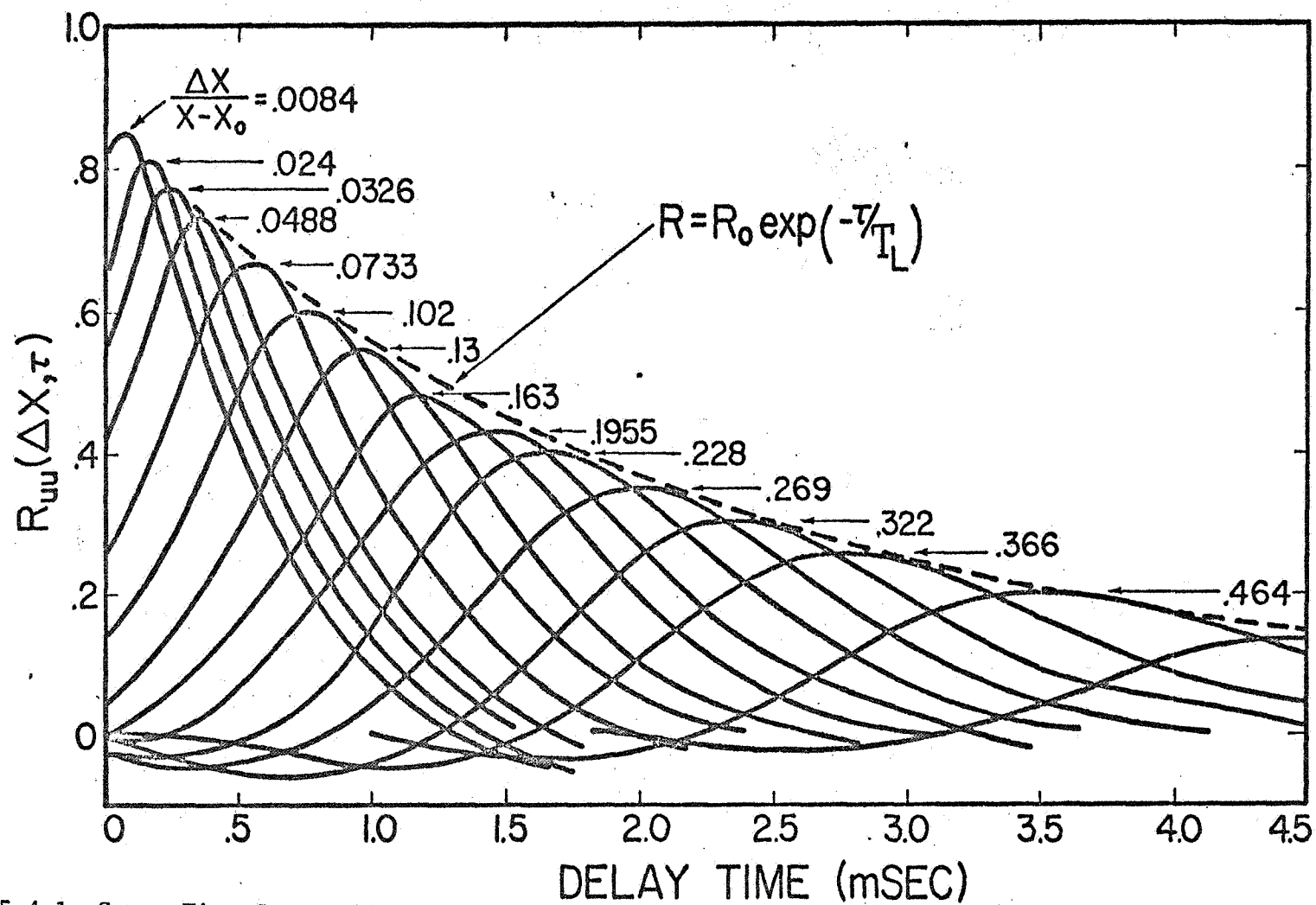


Fig. 5.4-1 Space-Time Correlation of Axial Velocity. $r/R = 1.0$, $x - x_0 = 15.4$ in., $U_j = 300$ ft./sec.

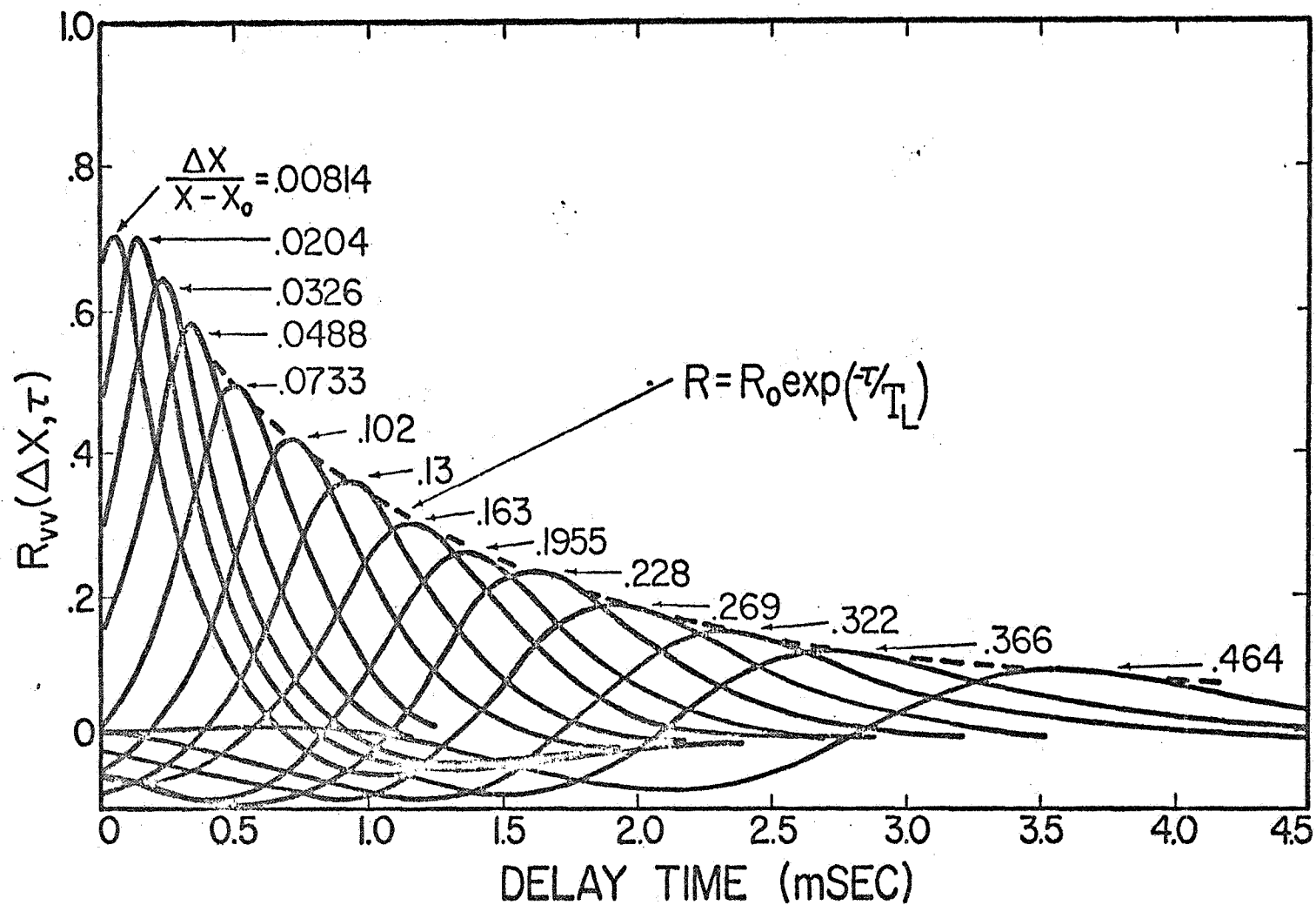


Fig. 5.4-2 Space-Time Correlation of Transverse Velocity. $r/R = 1.0$, $x/D = 4$, $U = 300$ ft./sec., $x = -0.4$ in.

maximized. This frame may be physically visualized and described by its auto-correlation--the envelope of measured axial space-time correlations. See Fig. 5.4-1 for an example. The time evolution of the convected turbulence is frequently described by the time scale $\mathcal{T}$. By standard definition, $\mathcal{T}$ is the area under the envelope. Convection velocity of the frame is determined from the position-time coordinates of the envelope and is given by

$$U_c = \Delta x / \Delta \mathcal{T} \quad (5-13)$$

The convected frame auto-correlations were found to be accurately represented, except for small separations, by the simple exponential relation, $R = R_0 e^{-(\mathcal{T}/\tau)}$. In this form, R_0 accounts for the necessary lateral offset and τ is identified as the convected frame integral time scale. The exponential nature of the correlation has been found by several other investigators including Davies et al. (1971).

To first order the convected frame time scale near the mixing center is a linearly-increasing function of axial position as shown in Fig. 5.4-2. This figure also clearly shows a second order effect damping the linear downstream increase. The damping is more severe for the transverse component.

Townsend's (1956) estimates of characteristic times indicates that the dominate coherence-destroying mechanism in heavily sheared flow is the mean shear deformation. A competing mechanism in regions of lower mean shear and higher turbulent intensity is turbulent-turbulent deformation. Restricting attention to the central high

shear mixing region, the dominance of mean shear deformation provides a basis for scaling τ_L in different flow regimes. Since a characteristic time for mean deformation is $t_s = (\frac{\partial \bar{u}}{\partial r})^{-1}$, the ratio $\tau_L / t_s = \tau_L |\frac{\partial \bar{u}}{\partial r}|$ could be expected to be a constant. Davies et al. (1963) and Wygnanski and Fiedler (1970) used this idea and found $\tau_L |\frac{\partial \bar{u}}{\partial r}| = 4.5$ near the center of mixing layers of varying geometries operated at several speeds. Our results for the u component of velocity in Fig. 5.4-3 and 5.4-4 are in general agreement with this value. The second order departure from a constant $\tau_L |\frac{\partial \bar{u}}{\partial r}|$ shown in Fig. 5.4-3 is probably due to the increasing effect of turbulent-turbulent coherence destruction with slowly increasing turbulent intensity levels downstream. Figure 5.4-4 shows the scaling is valid over a relatively wide portion of the central mixing region. However, again the decrease near the outer edges is probably due to increased prominence of turbulent-turbulent deformation compared to mean shear deformation.

An interesting side issue is that the deformation time basis for is apparently too restrictive for shear layers in general. Jones, Planchon and Hammersley (1973) found $\tau_L |\frac{\partial \bar{u}}{\partial r}| = 2.5$ near the mixing center of a plane two-stream mixing layer. The investigation of large scale structure by Jones, Hammersley, Planchon, and Spencer (1973), the shadow graphs of Brown and Roshko (1971), and the results presented by Kaplan (1973) suggest an alternate decorrelation length hypothesis as a scaling basis.

To explore this idea, it is hypothesized that the ratio of convected frame decorrelation length to mixing layer width is constant. With this assumption, and known characteristics of mixing layers a

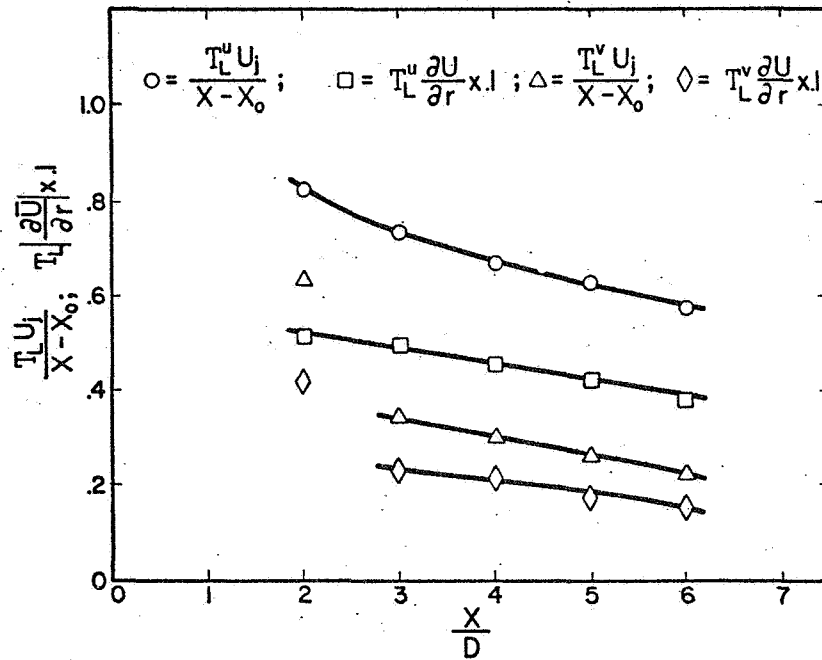


Fig. 5.4-3 Convected Frame integral Time Scale, $r/R = 1.0$.

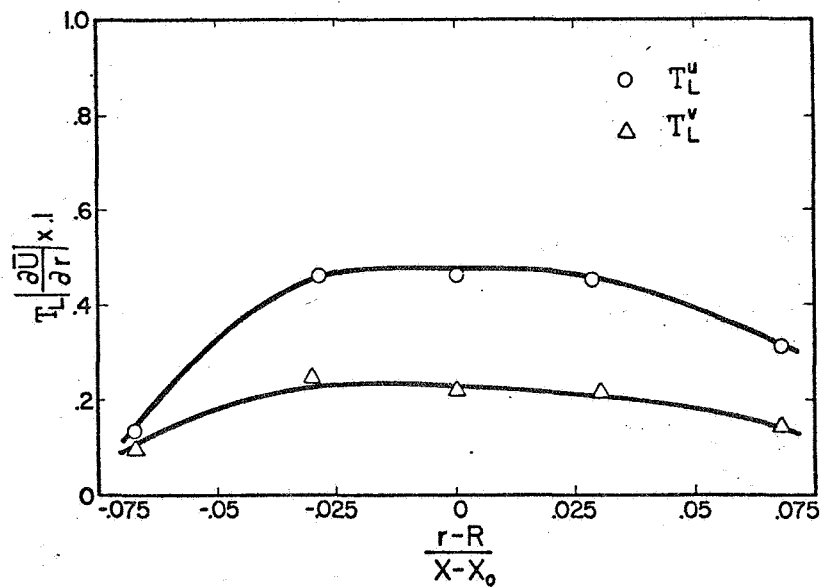


Fig. 5.4-4 Convected Frame Integral Time Scale, $x/D = 4$,
 $U = 300$ ft./sec., $R = 1.25$ in., $x = -0.4$ in.

generalized scaling law can be deduced. The convected frame decorrelation length scale is proportional to the decorrelation time scale times the velocity at which the convected frame moves. Restricting attention to the mixing centerline, we take the movement speed as the mean of the free stream velocities U_a and U_b . Then

$$L_L = T_L \frac{(U_a + U_b)}{2} = T_L \frac{U_a}{2} \left(1 + \frac{U_b}{U_a}\right) \quad (5-14)$$

where U_a and U_b are speeds of the high and low velocity free streams.

In a two-stream mixing layer the width b is proportioned as

$$\frac{U_a - U_b}{b} = C \frac{\partial \bar{U}}{\partial r} \quad (5-15)$$

where C is a constant. Together these relationships form

$$\frac{L_L}{b} = \frac{C}{2} T_L \frac{\partial \bar{U}}{\partial r} \left(\frac{1 + \frac{U_b}{U_a}}{1 - \frac{U_b}{U_a}} \right) \quad (5-16)$$

Equation 5-16 is the resulting generalized scaling law which may be tested by comparing it to experimental results. For a simple jet $U_b/U_a = 0$, thus

$$T_L \frac{\partial \bar{U}}{\partial r} \left(\frac{1 + \frac{U_b}{U_a}}{1 - \frac{U_b}{U_a}} \right) = 4.5 \quad (5-17)$$

For Jones, Planchon, and Hammersley's mixing layer at $\frac{U_b}{U_a} = 0.3$

$$T_L \frac{\partial \bar{U}}{\partial r} \left(\frac{1 + \frac{U_b}{U_a}}{1 - \frac{U_b}{U_a}} \right) = 2.5 \left(\frac{1.3}{.7} \right) = 4.64 \quad (5-18)$$

The agreement is fortuitously good considering the experimental errors possible. Admittedly, the hypothesis is largely intuitively based and lacks a mechanistic or analytical basis; however, it does seem to merit further work and experimental verification.

The convection velocities are determined by direct application of Eq. 5-13. To ensure accuracy, the values for U_c/U_j were found with a least squares fit. The standard error was 4% and it demonstrated little variation in U_c with separations and small experimental errors. The convection velocity was found to be almost constant on the mixing layer centerline as shown in Fig. 5.4-5. The transverse variation was found to be much less than the variation in mean velocity as shown in Fig. 5.4-6. Both observations agree with results reported by other experimenters.

The trends of convection velocity are explained with reference to the turbulence scales. Shear layer turbulence is generally held to be composed of a hierarchy of eddy sizes. Jones, Planchon, and Hammersley (1973) and several others have found larger eddies tend to convect at speeds characteristic of the average mixing layer speed. The smaller structure convects at velocities characteristic of the local material velocity. If the overall convection velocity is a spectrum-weighted average, the lack of variation in Fig. 5.4-6 results from the large eddy action. U_c^v is generally larger than U_c^u , both along the mixing center and across the layer. This difference is small and its difference is not clearly understood. Wills (1964) also noted a difference; however, his results show the distributions crossing near the centerline-- U_c^v variation being even flatter than U_c^u . A complete explanation for this

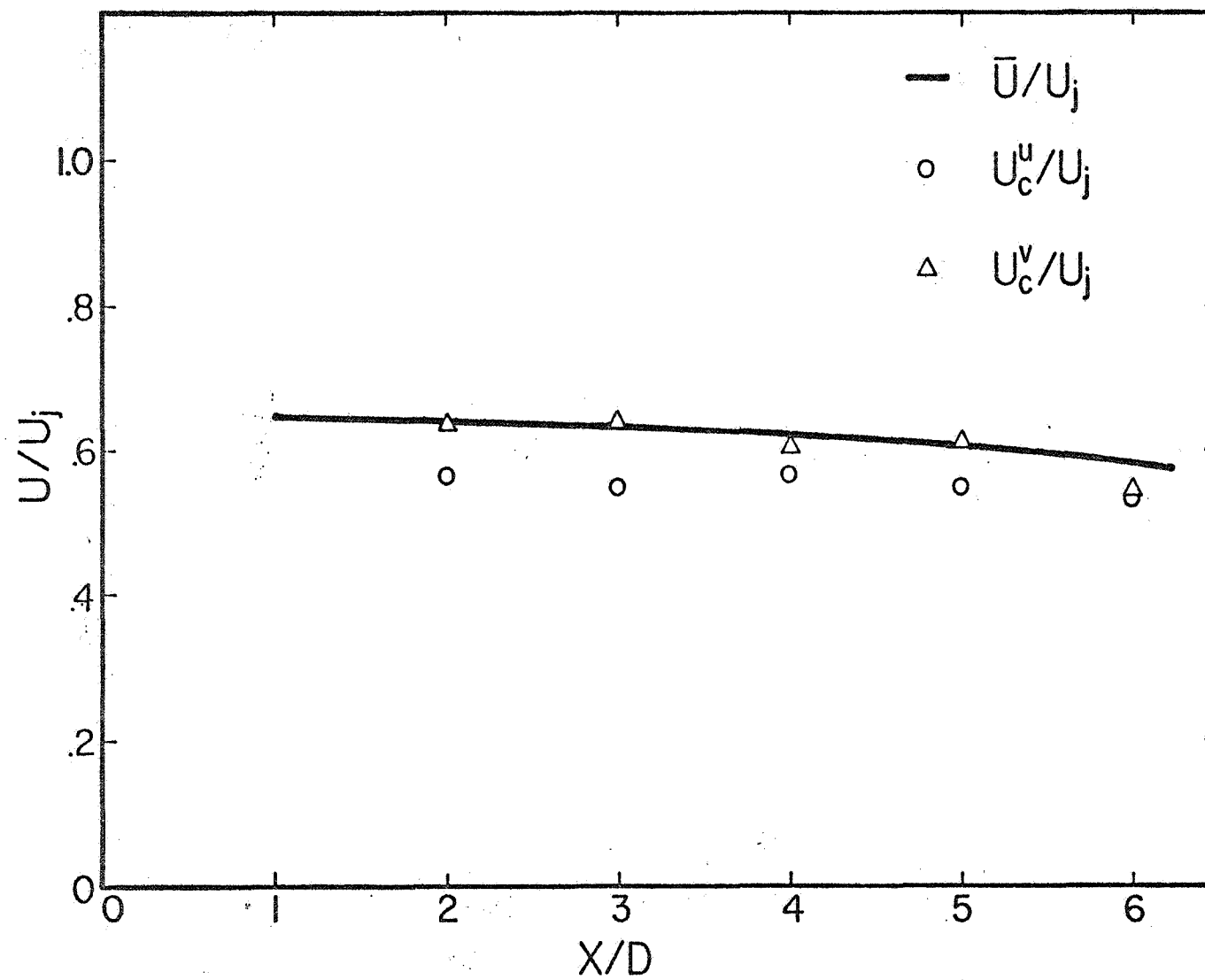


Fig. 5.4-5 Convection Velocity Axial Distribution. $r/R = 1.0$, $U_j = 300$ ft./sec, $D = 2.5$ in.

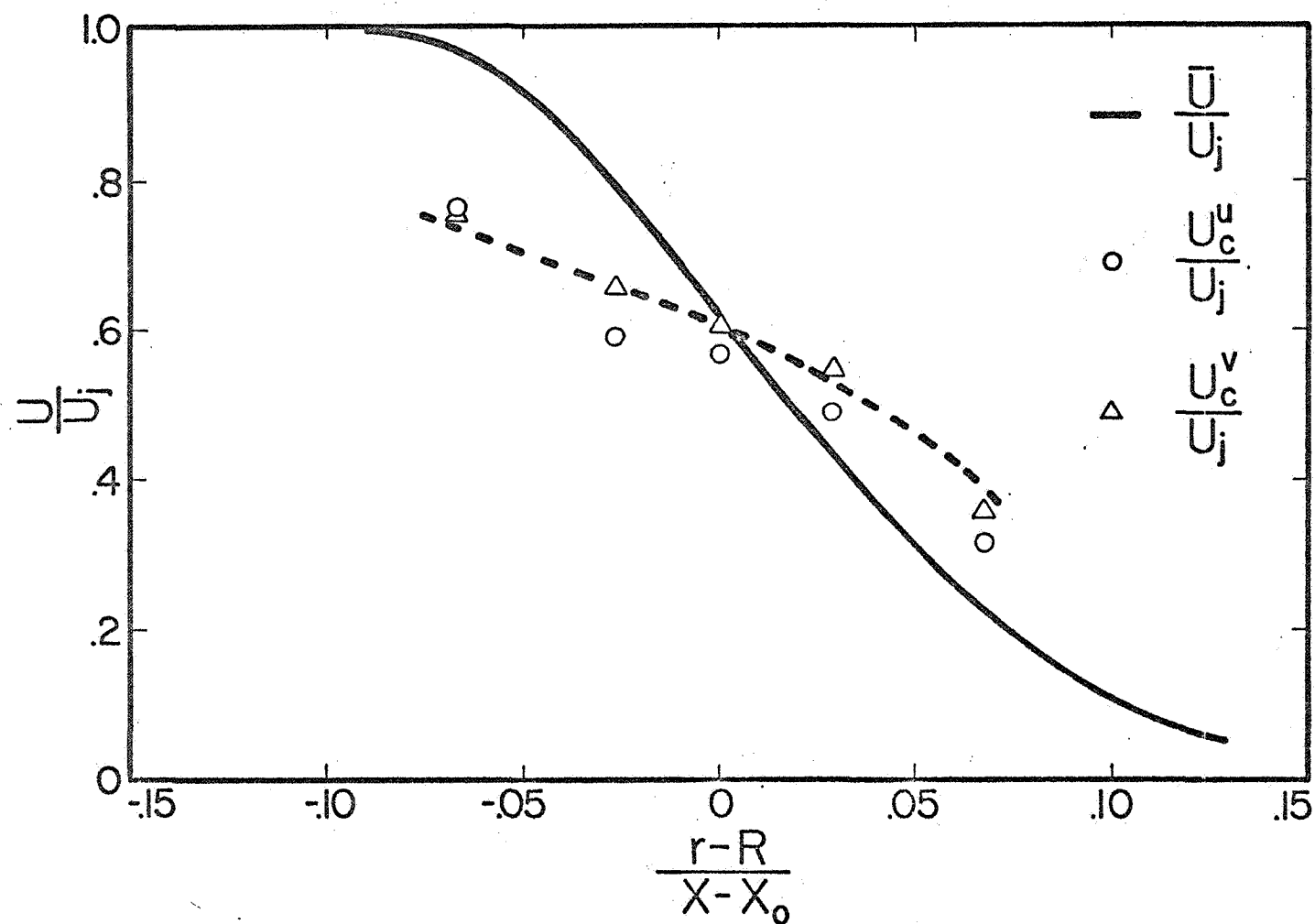


Fig. 5.4-6 Convection Velocity Transverse Distribution. $x/D = 4$, $U_j = 300$ ft./sec., $R = 2.5$ in.,
 $x_0 = -0.4$ in.

small difference is not available although it would seem to result from second order effects only slightly larger than the measurement errors involved.

5.4.2 Space Correlations

The space correlation is a special case of the general space-time correlation evaluated at $\mathcal{T} = 0$. It is a direct measure of the size and shape of turbulent coherent regions or "eddies". As shown in Chapter 2, this correlation is used directly in calculating pressure intensity.

The integral scale is a useful correlation length characterization. It is defined as the area under the space correlation and interpreted as the average dimension of the energy-containing eddies. The scale anisotropy of a flow, or the shape of the eddies, can be described by the scale anisotropy factors, which are the ratios of scales in different directions.

As we will see, scale interpretation can become ambiguous in non-isotropic flows. To establish a consistent interpretation, we turn to the well-established theory of isotropic turbulence. An additional reason for this background is that modified isotropic forms will be used to model the non-isotropic space correlations.

In homogeneous isotropic turbulence, the general two-point correlation can be expressed in terms of two scalars f and g . Following Hinze (1959), f and g are correlations of velocity components parallel and perpendicular to separation, respectively. For incompressible flows, g can be expressed in terms of f . This allows prediction of the full velocity space correlation in terms of one

measured scalar function. Additionally, integral scales determined from f and g are related, f being twice as large as g . Generalizing, our guide for scale comparison and anisotropy determination is clear--compare correlations and scales only for velocity components parallel to the separations. Several investigators have measured scales from $R_{uu}(\Delta x)$ and $R_{uu}(\Delta r)$ reporting $L_x^u/L_y^u = 2$, a result one would expect in isotropic turbulence. Obviously this does not imply eddies are elongated in the axial direction.

Consistent with this guideline, we focus upon the correlations $R_{uu}(\Delta x)$, $R_{vv}(\Delta r)$ and $R_{ww}(\Delta z)$ to establish scale variation, scale anisotropy, and the functional form of the correlations. The correlations were fortunately found to be represented quite well by an exponential $R = R_0 e^{-\Delta x/L_x}$ as shown in Fig. 5.4-7. Consequently, even with the probe offset the scales could be examined as the parameter in an exponential fit.

Both L_x^u and L_r^v were found, as expected, to vary linearly with axial distance. A small variation with transverse position was found. The scale variations shown in Fig. 5.4-8 and 5.4-9 are consistent with those of Davies, Fisher, and Baratt (1963); Wignanski and Fiedler (1970); Lawrence (1956); and others. The scale anisotropy may be evaluated as the ratio of scales. It was found to be

$$L_r^v/L_x^u = 0.9 \quad ; \quad L_z^w/L_x^u = 0.55 \quad (5-19)$$

Bradshaw et al. (1964) determined similar scales by finding the distance at which the correlation function decreased to 0.5. In comparison, his anisotropy ratios may be computed as:

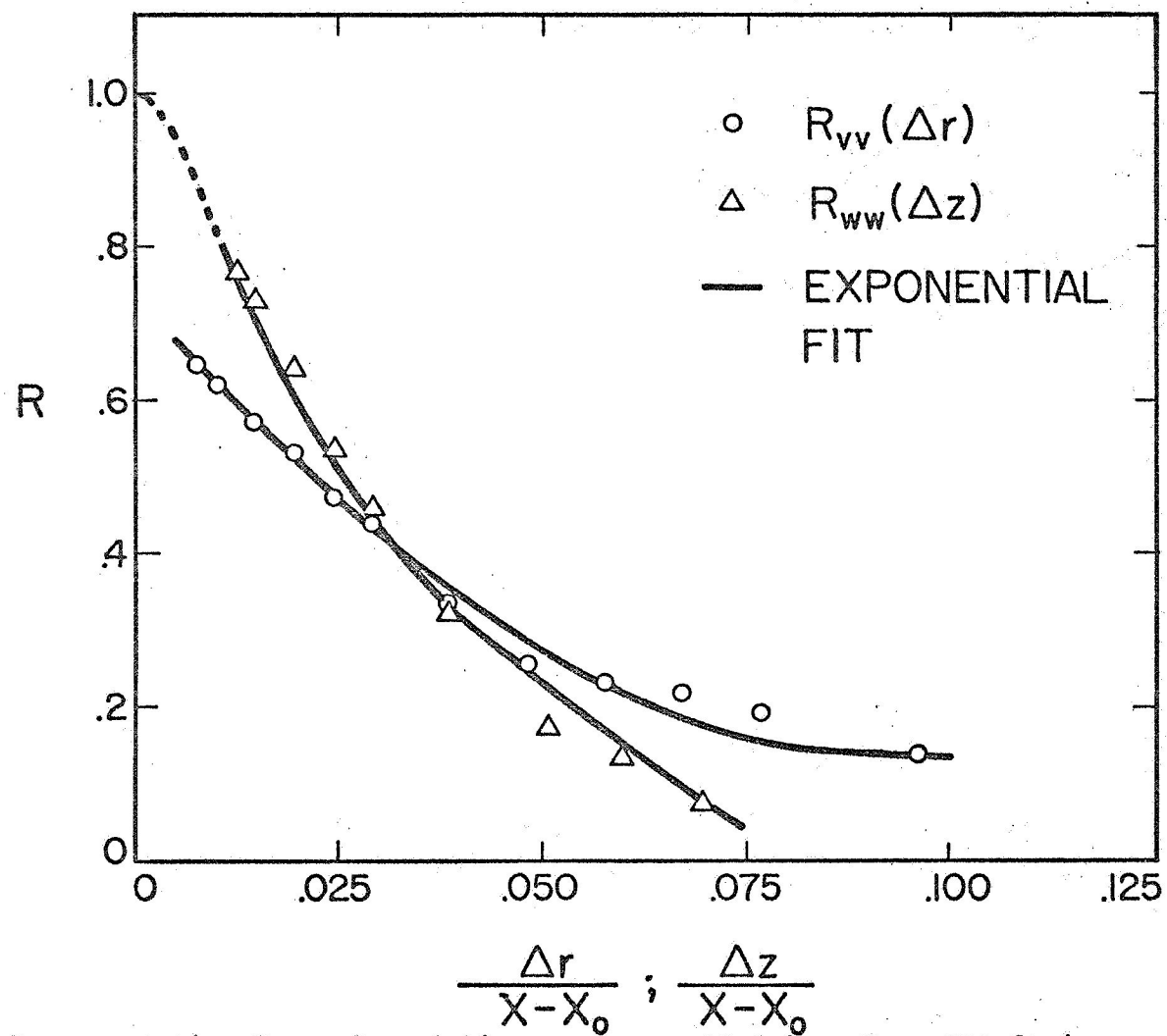


Fig. 5.4-7 Representative Space Correlations. $x - x_0 = 10.4$ in., $U_j = 300$ ft./sec.

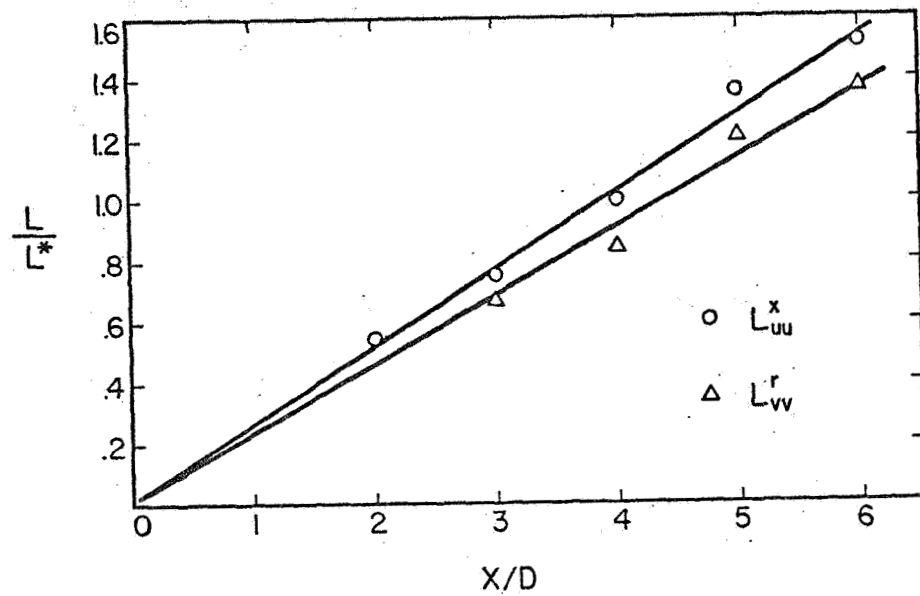


Fig. 5.4-8 Space Scales Axial Distribution. $r/R = 1.0$.

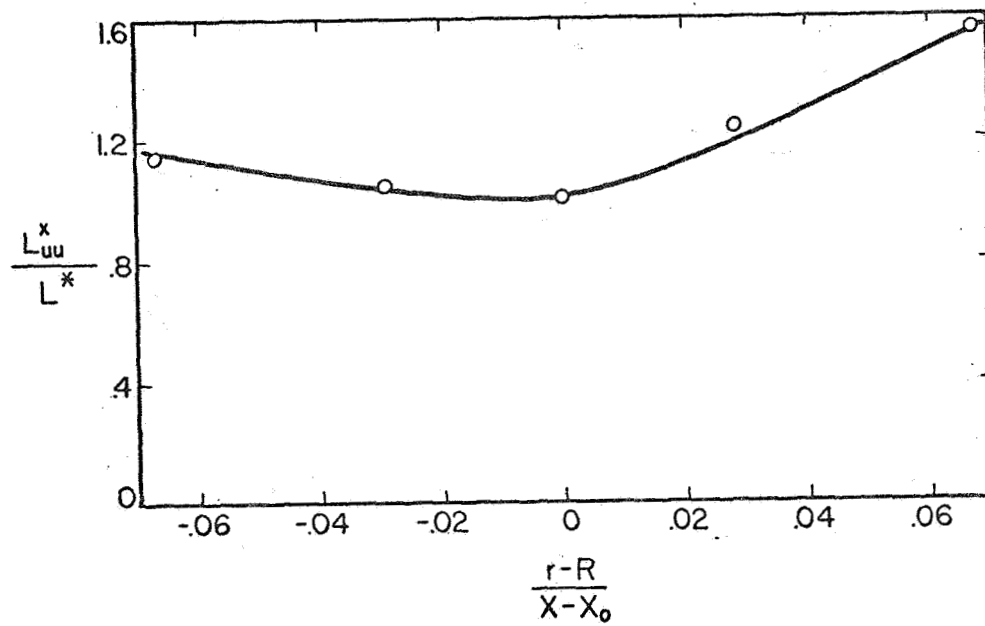


Fig. 5.4-9 Space Scales Transverse distribution. $x/D = 4$,
 $U = 300$ ft./sec., $R = 1.25$ in., $x = -0.4$ in.

$$L_r^v / L_x^u = 0.7 \quad ; \quad L_z^w / L_x^u = 1.1 \quad (5-20)$$

Their x probe interference or offset is not discussed; thus, it is difficult to explain the difference.

In summary, the space correlations and scale variations were found to be in close agreement with those of Lawrence (1956); Davies, Fisher, and Baratt (1963); and Wignanski and Fiedler (1970). Specifically in the mixing region:

- a. Scales grow linearly,
- b. Scales vary slowly with transverse direction,
- c. Axial-transverse scale anisotropy is about 0.9 and
- d. Correlations are well represented by an exponential except near zero separation.

5.5 Elongated Isotropic Forms as a Space Correlation Model

A model of the velocity space-time correlation is necessary for predicting the pressure intensity. As discussed in Chapter 2, one such model is Kraichnan's (1956) scale anisotropy model. In review, this model hypothesizes an isotropic turbulence structure elongated in the flow direction by scale factor α . The implications of such a model are that velocity intensities and scales are related by the same anisotropy factor $\alpha = \frac{L_x^u}{L_r^v} = \frac{u'}{v'}$. Further, once L_x^u , α and one scalar analogous to the isotropic f function is determined, the general velocity correlation may be inferred. In addition to the fundamental objections to the model, detailed comparisons showed it

was inadequate as a representation of the full velocity space-time correlation. However, it was found that the results of the model, if used in a narrow empirical sense, could be employed to represent R_{VV} . Since R_{VV} is the major pressure source, this representation was pursued.

As was shown in Chapter 2, for isotropic conditions R_{VV} may be written as

$$R_{VV} = f(\xi) + \frac{1}{2} \xi f'(\xi) - \frac{1}{2} f''(\xi) \frac{\xi^2}{\xi} \quad (5-21)$$

applying the elongation transformation

$$\Delta X = \Delta_1 \xi_1, \quad \Delta r = \xi_2, \quad \Delta Z = \xi_3 \quad (5-22)$$

and assuming the defining scalar function is the exponential $f = e^{-\sigma \xi}$ results in the form for R_{VV} of

$$R_{VV} = e^{-\sigma \left(\frac{(\Delta X)^2}{\Delta_1^2} + \Delta r^2 + \Delta Z^2 \right)^{1/2}} \left[\frac{1 - \frac{\sigma}{2} \left(\frac{(\Delta X)^2}{\Delta_1^2} + \Delta r^2 + \Delta Z^2 \right)^{1/2}}{+ \frac{1}{2} \sigma \Delta r^2} \right] \quad (5-23)$$

This equation is the same as Eq. 2-31 which was used in the pressure theory development. The constants σ and Δ_1 in Eq. 5-23 were determined by fitting the experimental data for $R_{VV}(\Delta r, \epsilon_z)$ (Fig. 5.5-2) and $R_{VV}(\Delta X, \epsilon_z)$ (Fig. 5.5-2). It should be noted that this form allows one to account for the probe offset ϵ_z . As shown in Fig. 5.5-1 and 5.5-2, the functional form seems to represent the correlation well,

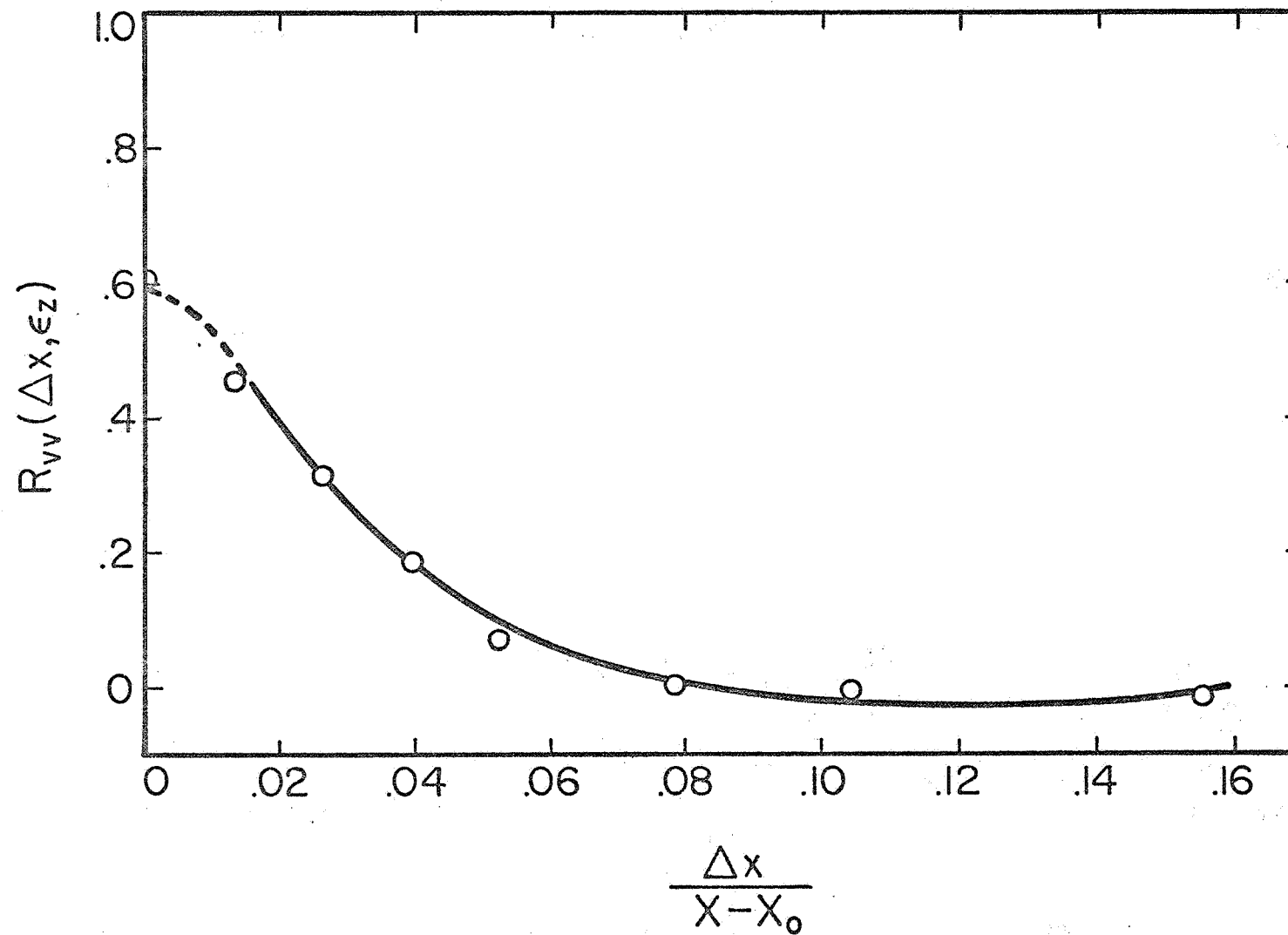


Fig. 5.5-1 Elongated Isotropic Model for Axial Separations. Points are Data, Line is Model. Stationary Sensor at $x/D = 4$, $r/R = 1.0$, $U = 300$ ft./sec., $x = -0.4$ in.

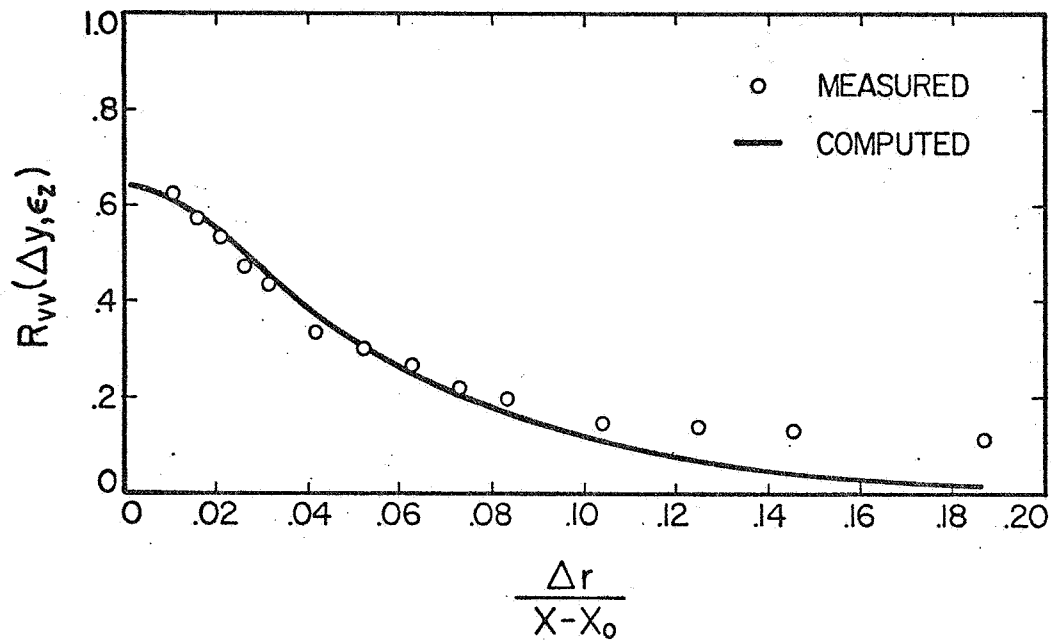


Fig. 5.5-2 Elongated Isotropic Model for Transverse Separations.

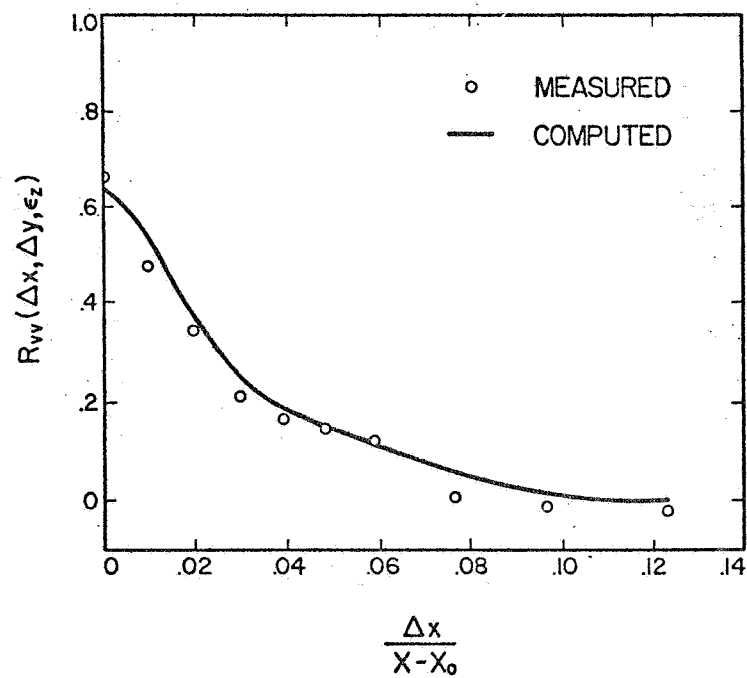


Fig. 5.5-3 Elongated Isotropic Model for Diagonal Separations.
 Separations on line 30° from Axial. Stationary Sensor at
 $x/D = 4$, $r/R = 1.0$, $U = 300$ ft./sec.

particularly at separations near one integral scale length where the maximum contribution to the pressure intensity is made. The model passes the real test of predicting R_{vv} for both Δx and Δr separations. Figure 5.5-3 shows the predicted and measured values of $R_{vv}(\Delta x, \Delta y)$ for a transverse about 30° from the downstream direction. The jet parameters determined in Fig. 5.5-1 and Fig. 5.5-2 and used in Fig. 5.5-3 are $1/\sigma(x-x_0) = .044$ and $\mathcal{A}_1 = 0.88$.

In conclusion, Kraichnan's elongated isotropic forms did not represent the general space-time correlation. For an example of the ambiguity, the general model predicts $\frac{L_x^q}{L_y} = \frac{u'}{v'} = \mathcal{A}_1$, but the values were found to be 1.18, 1.33, and .88, respectively. This range of values caused large differences between predicted and observed $R_{u_i u_j}$. It was found, however, that the elongated isotropic forms could be applied to R_{vv} in a narrow empirical sense and that this model agreed well with experimental results. Thus, a model of R_{vv} , necessary for predicting pressure intensities, is given by Eq. 5-23 where the values of the parameters are: $1/\sigma(x-x_0) = .044$ and $\mathcal{A}_1 = 0.88$.

6. FLUCTUATING PRESSURE FIELD EXPERIMENTAL RESULTS AND DISCUSSION

The objective of the pressure measurement program was to statistically characterize the mixing layer fluctuation pressure field. The characterization is in terms of quantities describing intensity, size of correlated regions, and the convection and evolution of the correlated regions. The measurements were also designed to provide data for an important practical problem; namely, to provide pressure space-time correlations for the Ribner (1964) dilatation model of aerodynamic noise generation. The results are divided along experimental lines into a one-point and a two-point measurement section.

In the presentation of the results, emphasis has been placed upon scaling laws. This is important for both model development and for understanding the physical processes involved. Guidance for the point intensity scaling follows from the theory in Chapter 2. The theory for two-point results is insufficiently developed to provide comparable explicit guidance. Consequently, scaling suggestions have been taken from dimensional reasoning and intuitive comparisons with the well-established velocity field scaling laws.

6.1 Point Pressure Distributions

6.1.1 Intensity

The measured radial distributions of pressure intensity are shown in Fig. 6.1-1 and Fig. 6.1-2. Although these distributions are similar in shape, the peak values are significantly different. The build-up in the developing mixing region and decay in the transition

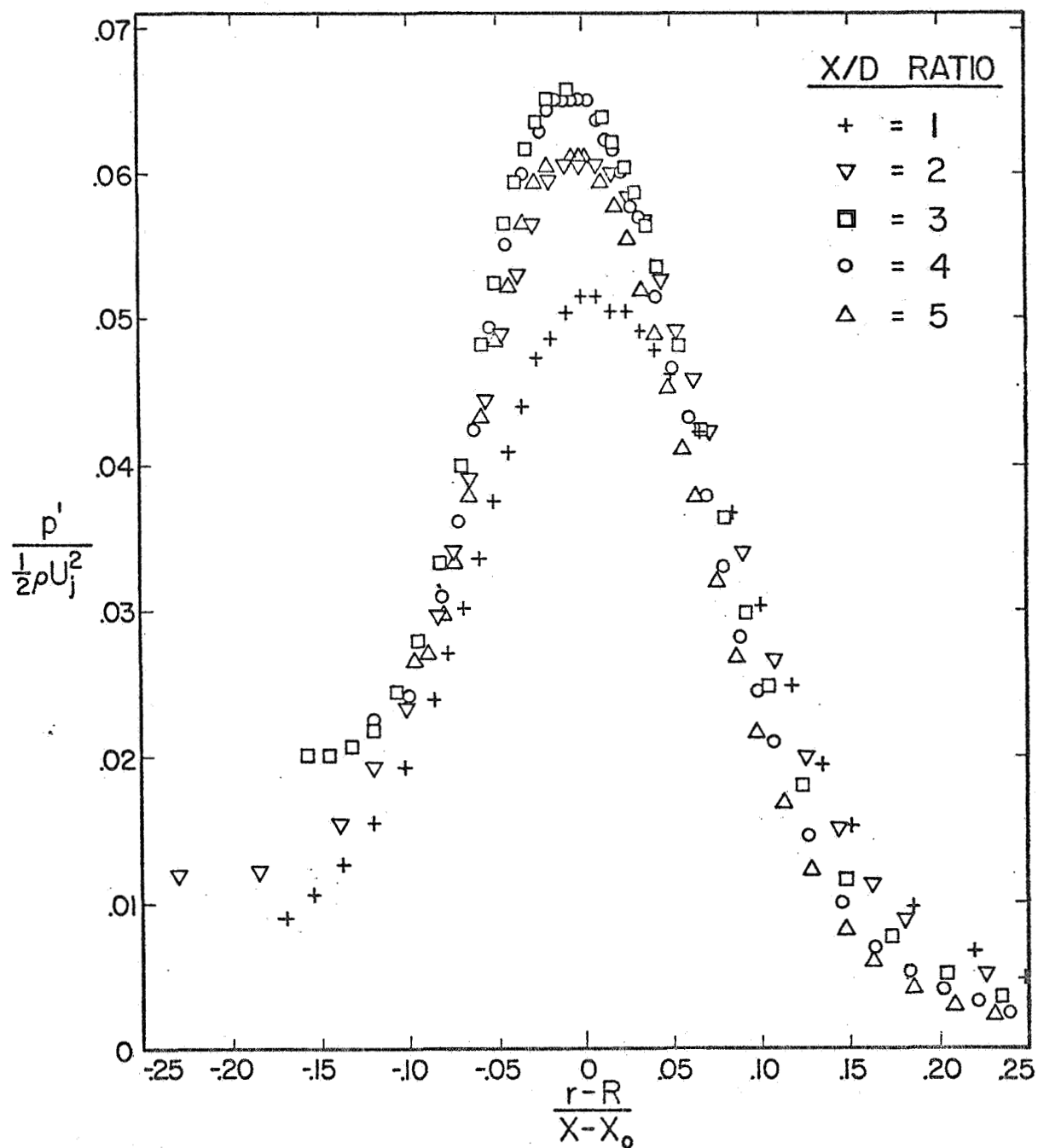


Fig. 6.1-1 Pressure Intensity, Driven Mixing Region. $U_j = 300$ ft./sec.,
 $R = 1.25$ in., $x_0 = -0.4$ in.

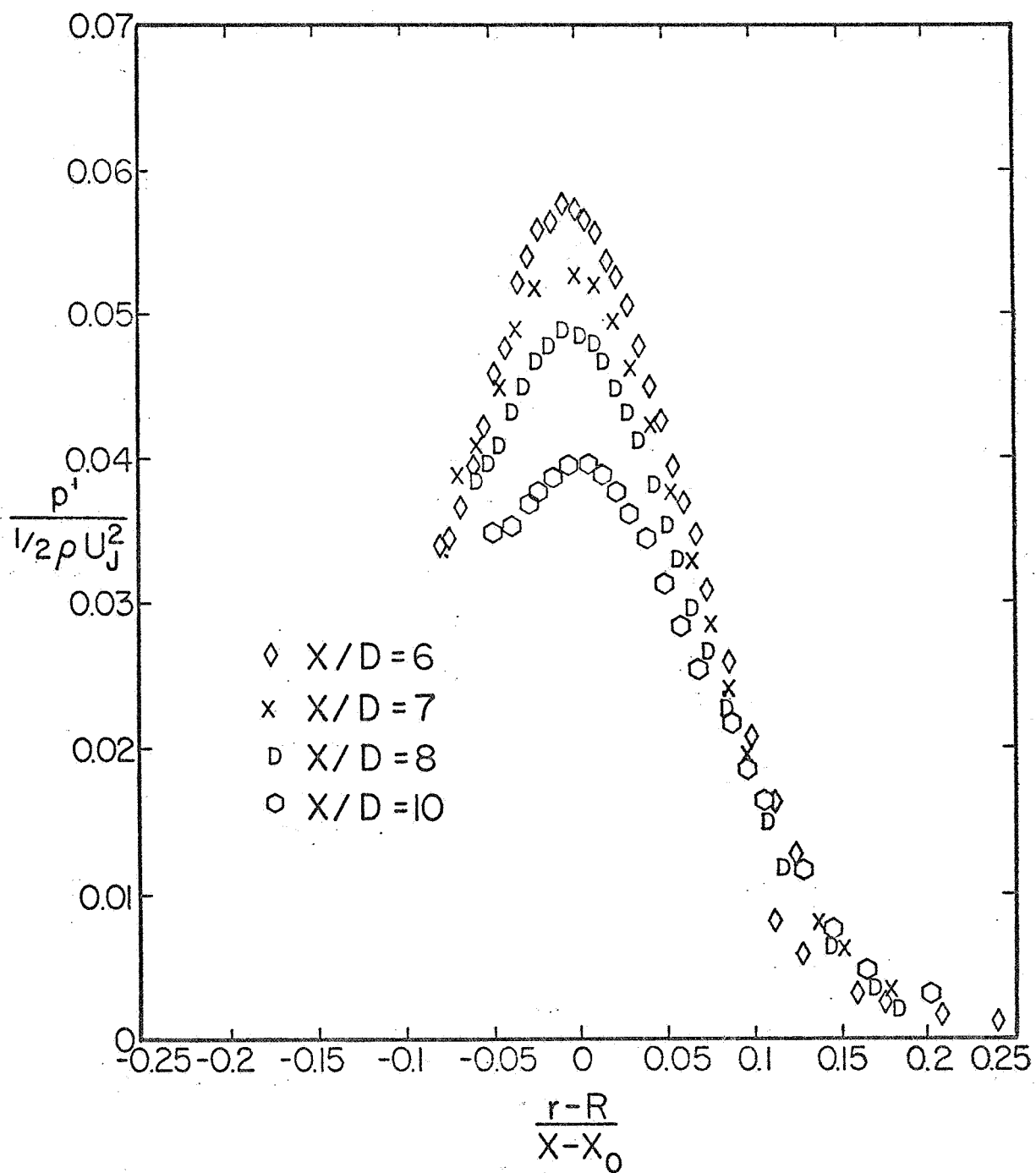


Fig. 6.1-2 Pressure Intensity, Transition Region. $U_j = 300$ ft./sec.,
 $R = 1.25$ in., $x = -0.4$ in.

region will be explained with reference to the theory developed in Chapter 2.

The general results of the theory for the turbulence-mean shear contribution was given in Eq. 2-34. Recast in terms of the quantities measured and previously presented, the equation is

$$\frac{P_{TM}}{\frac{1}{2}\rho U_j^3} = 2 \sqrt{\frac{8}{15}} \left[\left[\frac{2(U/U_j)}{2r} (x-x_0) \right] \left[\frac{V'}{U_j} \right] \left[\frac{Lr}{x-x_0} \right] \sqrt{J(\lambda) \frac{2+\lambda^2}{3}} \right] \quad (6-1)$$

Considering the many simplifying assumptions necessary in the development of Eq. 6-1, Fig. 6.1-3 shows remarkably good agreement between measured and predicted intensity distributions. To predict the intensity distribution, the gradient factor was taken from Fig. 5.2-3, the intensity from Fig. 5.3-2 and Fig. 5.3-5, and the structural factor was assumed constant. The scale factor was taken as a constant in the mixing layer ($x/D < 5$), as shown in Fig. 5.3-9. In the transition region scales were not measured; therefore, they were assumed to increase linearly with axial position but at half the rate as in the mixing layer. This assumption is suggested by Lawrence (1956), Davies et al. (1963), and Nayar et al. (1969).

The calculated and experimental distributions are normalized by their peak value to show variation rather than absolute value. At $x/D = 4$, the calculated intensity absolute value was 17% higher than that measured. The difference reflects the assumption of slowly varying mean velocity gradient and velocity intensity in Eq. 2-15, and uncertainty in experimentally determining the several velocity field parameters in Eq. 6-1. The difference cannot be explained as the

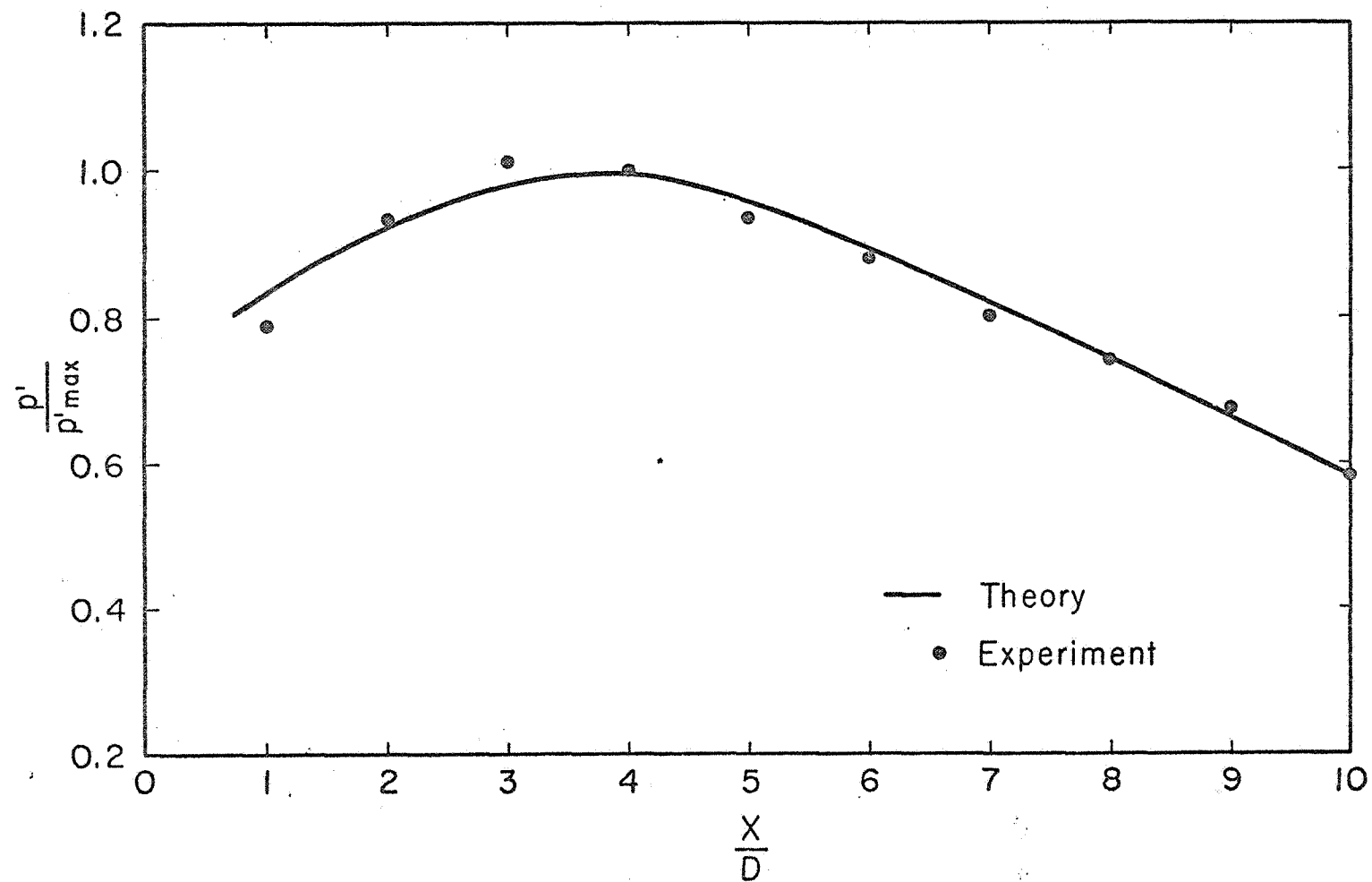


Fig. 6.1-3 Comparison of Theoretical and Experimental Pressure Intensities. $r/R = 1.0$,
 $U_j = 300$ ft./sec.

contribution from P_{TT} or pressure transducer errors from the velocity field as both would produce larger measured pressure intensities.

The variations of the intensity may be traced via. Eq. 6-1 to specific velocity field variations. In the mixing region only the factor v'/u_j in Eq. 6-1 has significant variation. This is the primary cause of the gradual pressure intensity increase. In the transition region v'/u_j stabilizes and the almost linear decrease of pressure intensity is precipitated by decreases in both the gradient factor and the scale factor.

A comparison of calculated P_{TM} and experimental pressure intensity distributions across the mixing layer is shown in Fig. 6.1-4. The measured intensity corrected for velocity error is also shown to demonstrate the small error. The comparison between theoretical and actual values shows the limitations of the theory. In the central mixing region, $\left| \frac{r-R}{X-X_0} \right| < .05$, the assumption of slowly varying gradient and intensity (Refer to Eq. 2-15) is limiting. This assumption replaces volume-weighted intensities and gradients with point values and produces the calculated peaked, narrow intensity distribution. In the outer regions, $\left| \frac{r-R}{X-X_0} \right| > 0.1$, two other assumptions become questionable. First, in this region of small mean velocity gradient, the dominance of P_{TM} over P_{TT} can no longer be claimed. Second, in this region of low gradient velocity intensities the dominance of inertial over compressional effects is questionable. Certainly in the extreme, the near field is a compressional or acoustic pressure mode. From a theoretical viewpoint, the fundamental assumption of incompressibility is questioned for the outer region. It seems a more proper description

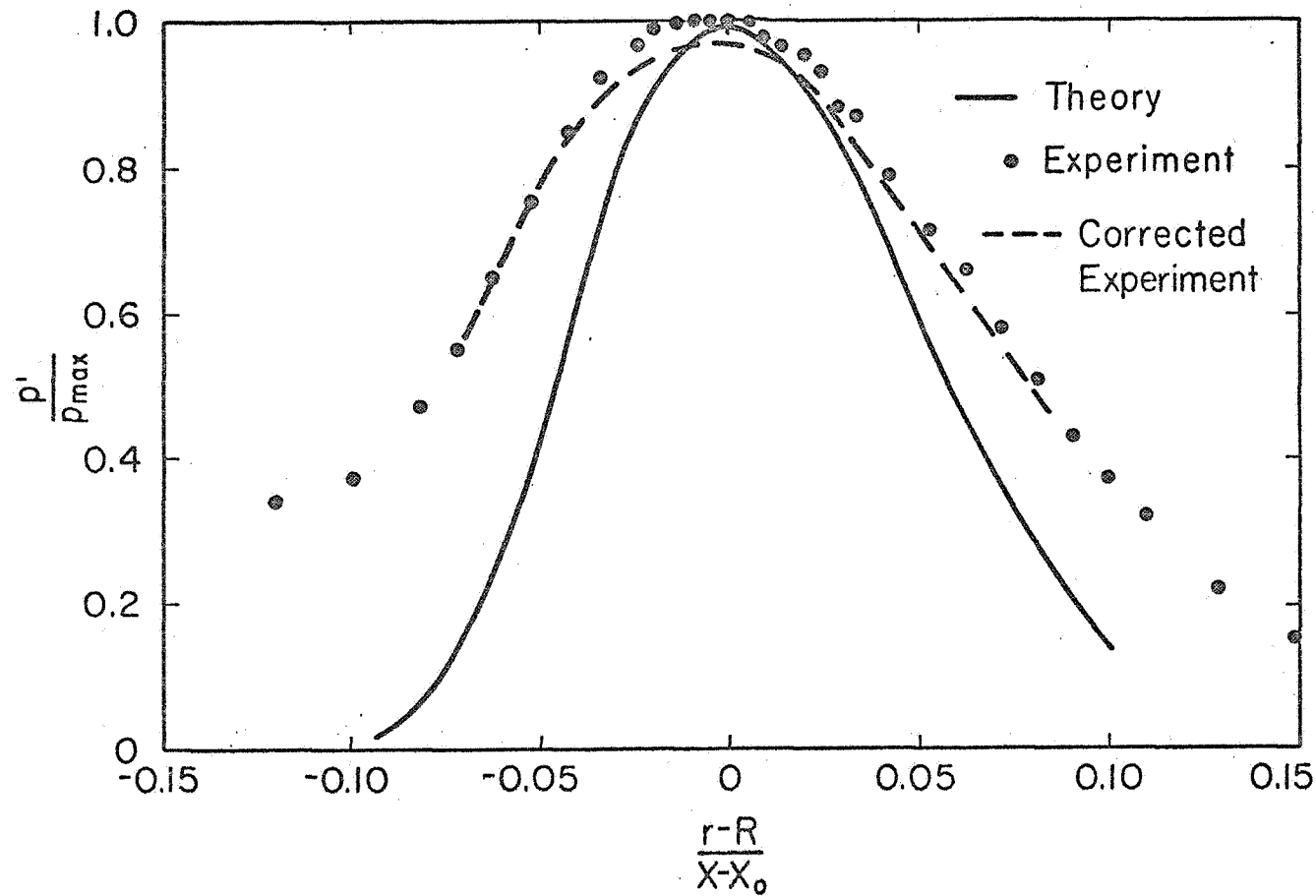


Fig. 6.1-4 Comparison of Theoretical and Measured Pressure Intensities $x/D = 4$. $U_j = 300$ ft./sec.,
 $R = 1.25$, $x_o = -0.4$ in.

could be made with the compressible linear momentum and continuity equations. This would result in a wave equation for pressure instead of the Poisson equation in Eq. 2-3. Thus, one concludes a totally different mechanism governs the outer region and the theory developed in Chapter 2 does not apply.

The pressure intensities on the jet centerline are shown in Fig. 6.1-5. Ko and Davies (1971) proposed that the pressure intensity in the jet core region is an irrotational acoustic manifestation of the shear region source. They further argued that since a hot wire measured fluctuating mass flux, their hot wire intensity measurements in the core were proportional to the fluctuating pressure. This follows because the anemometry would sense both fluctuations of density and displacement rate (velocity) of a pressure wave. Lau and Fuchs (1972) disagree with part of Ko's reasoning, but nevertheless agree that pressure and velocity intensities are proportional. The comparison between pressure and velocity intensities in Fig. 6.1-5 is consistent with these arguments for $1 < x/D < 3$. Beyond $x/D = 3$, the core center contains an increasing proportion of turbulence. Hence the velocity fluctuations cannot be identified solely as pressure perturbations.

6.1.2 Pressure Intensity Scaling

A hierarchy of pressure intensity scaling laws was developed in Chapter 2. A verification and error analysis of these laws for distinct system parameter variations would enhance their utility in engineering applications. Four of the more important parameters for which pressure intensity scaling should be studied are: (1) spatial position within

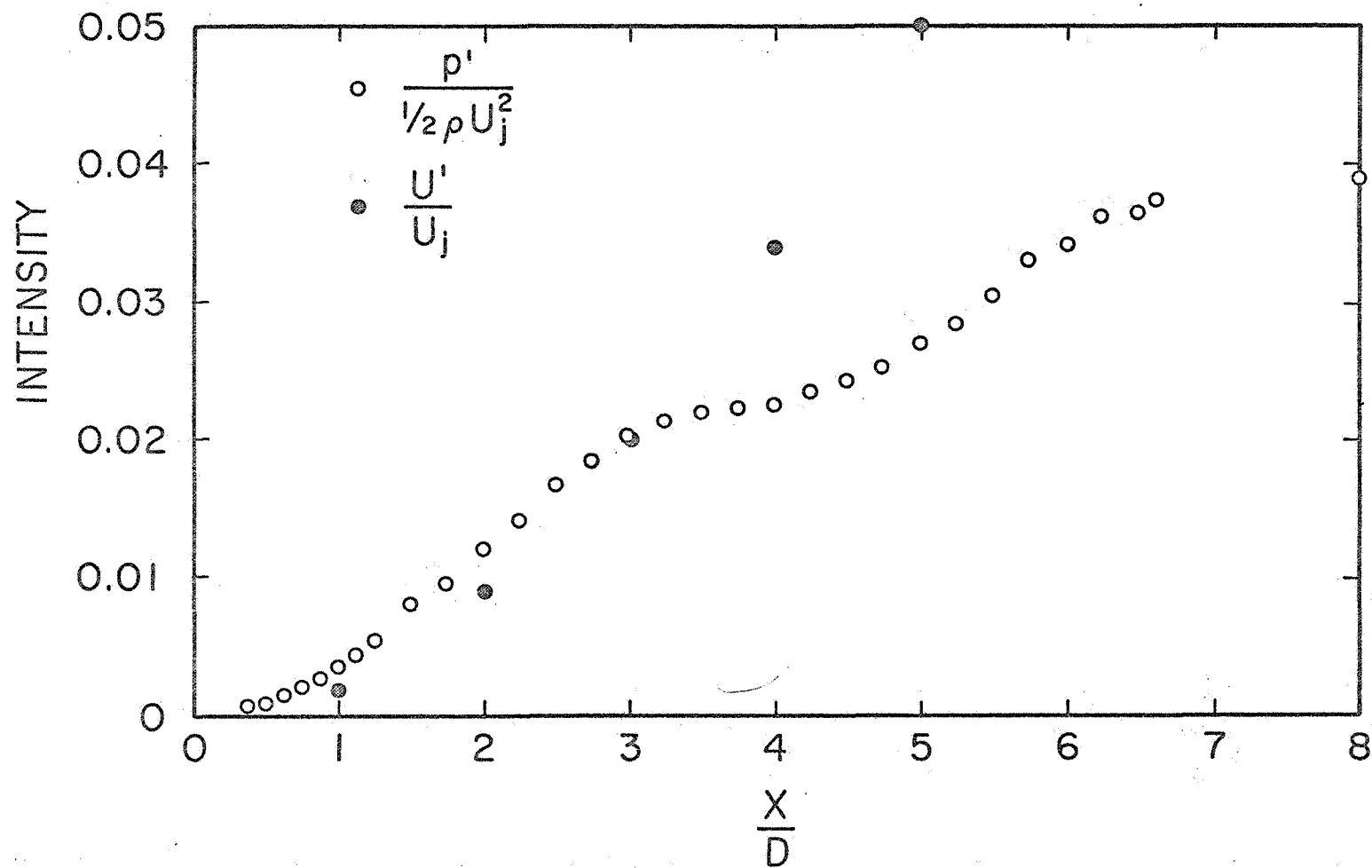


Fig. 6.1-5 Jet Centerline Pressure and Velocity Intensities. $r = 0$, $U_j = 300$ ft./sec., $D = 2.5$ in.

a given system, (2) system size (jet diameter), (3) system speed (jet exit velocity), (4) system velocity ratio.

Although the result of a scaling study is to approximate with limited information, the study ideally requires detailed data to identify and quantify the approximation errors. The first variation above has been studied in detail; however, only limited information is available to describe system size and speed effects and no information is yet available to check velocity ratio scaling.

An evaluation of peak pressure intensity variation for various spatial positions follows from Fig. 6.1-4. As was previously noted, variations in mean velocity gradient, transverse velocity intensity, and velocity scale accurately predict the variation of peak intensities in the mixing and transition region. If attention is restricted to the mixing region, the same high degree of accuracy of prediction is attained by accounting only for transverse velocity fluctuations. If no turbulent structure is used, errors are bounded by about 20%.

The following table summarizes these scaling laws.

Table 6.1-1 Pressure Intensity Scaling for Various Spatial Positions

<u>Scaling Law</u>	<u>Range of Applicability</u>	<u>Accuracy</u>
$\frac{p'_{pk}}{\frac{1}{2} \rho U_j^2 \left[\frac{\partial u}{\partial r} (x-x_0) \right] \left[\frac{v'}{U_j} \right] \left[\frac{L_v}{x-x_0} \right]} = \text{const.}$	Mixing and Transition Region	5%
$\frac{p'_{pk}}{\left[\frac{1}{2} \rho U_j^2 \right] \left[\frac{v'}{U_j} \right]} = \text{const.}$	Mixing Region	5%
$\frac{p'_{pk}}{\frac{1}{2} \rho U_j^2} = \text{const.}$	Mixing Region	20%

Evidence from a wide spectrum of experiments suggests the scaling laws should be valid for any size of jet. To first order it is generally held that mixing layer mean velocity gradient varies as $1/x$, turbulent scales vary as x and non-dimensional turbulent structure and turbulent intensities are size invariant. A rough comparison between a 1 1/2 in. diameter and 2 1/2 in. diameter jet at about 300 fps is shown in Fig. 6.1-6 and is consistent with this idea. The 1 1/2 in. jet results are preliminary; consequently, no great experimental accuracy can be claimed, and a detailed velocity field structure is not available. The 15% difference in scaled results could be reasonably expected as the result of small differences in mixing layer development. Further investigation of size scaling is clearly desirable. It seems future work in this area should focus upon and isolate the effect of mixing layer curvature, if any, upon the development of a shear layer. The mixing layer development could then be traced via mean velocity gradient, velocity intensity, scale, and structure directly to the pressure intensity.

As with jet size, the first order velocity field scaling laws predict $\rho^{1/2} \rho U_j^2 = \text{constant}$ for the mixing layer. Some experimental evidence, however, indicates that varying speed produces significant second order effects. Lawrence's (1956) results show velocity intensity decreasing with speed. Davies et al. (1971) found $\frac{\partial(u/u)}{\partial r}(x-x_0)$ largely constant; however, further refinements would be helpful. Variations of these quantities and consequently of pressure intensity with jet speed could be caused by two mechanisms. First, Reynolds number changes to the lip boundary layer could alter mixing layer transition and the

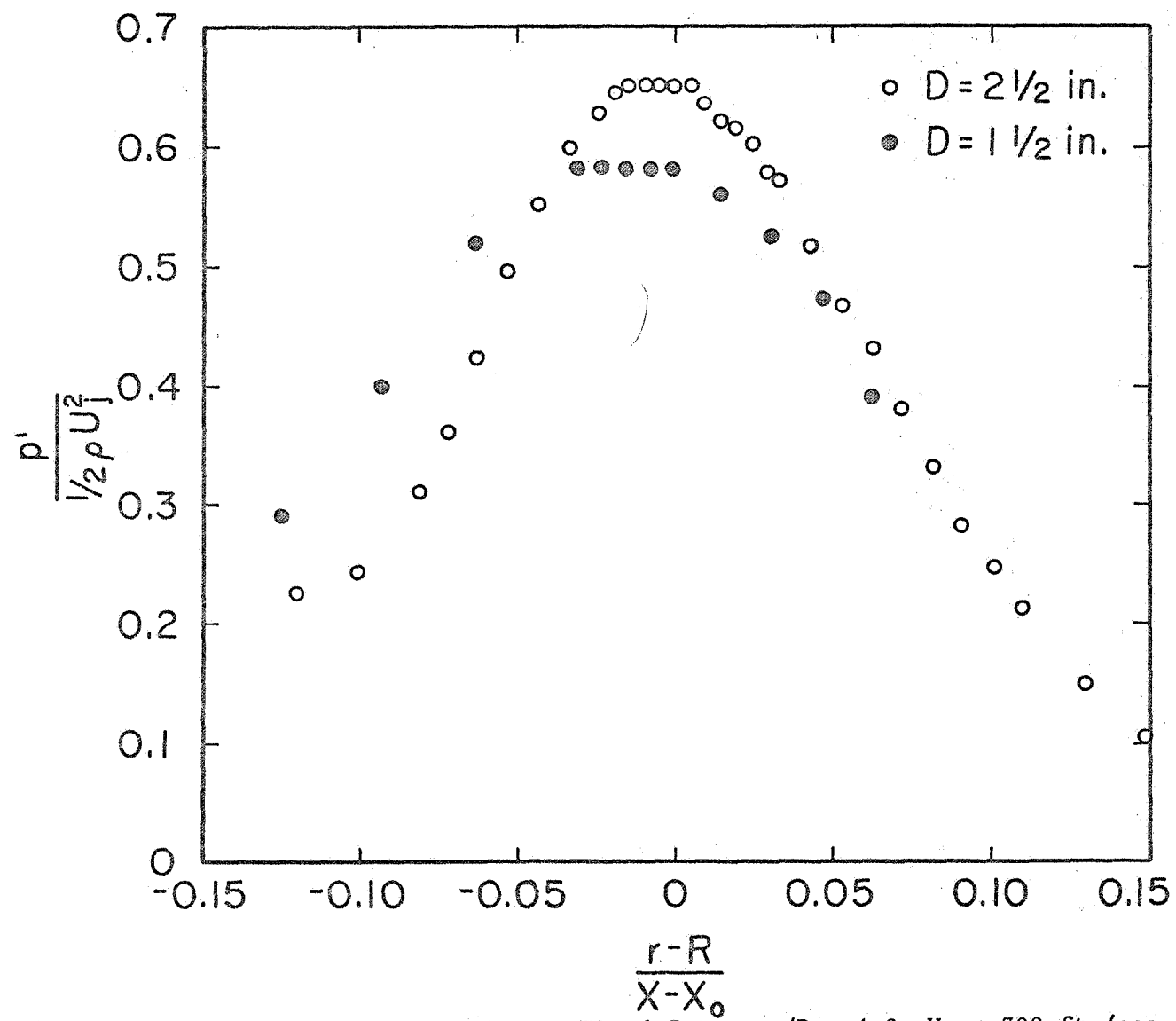


Fig. 6.1-6 Pressure Intensities for Different Sized Jets. $x/D = 4.0$, $U_j = 300$ ft./sec.

ensuing development. Second, compressibility, or Mach number effects, could change the mean velocity gradient, velocity intensities, and perhaps to a limited degree, scale and structural development. In the extreme, near or beyond $M = 1$, the applicability of the fundamental incompressible basis for the pressure theory must be questioned.

The first order scaling relation $\rho' / \frac{1}{2} \rho U_j^2 = \text{constant}$ was found to be within 15% in the incompressible range. Fig. 6.1-7 shows these results at $x/D = 4$ on the mixing center for the 2 1/2 in. jet operating from 100 to 375 feet per second. Velocity field data is not available to explain the gradual build-up at low velocities; however, it could result from minor changes the way the mean velocity gradient and turbulent intensity approach similarity.

Unfortunately, the flow facility capacity did not permit higher speeds to investigate scaling in the compressible range. Some very crude tests were made on Lawrence's (1956) jet facility at NASA Lewis Research Center. Notwithstanding possible errors, these tests suggested that $\rho' / \frac{1}{2} \rho U_j^2$ was constant up to 500 feet per second.

In summary, the experimental results show the scaling laws have merit for round jet mixing layers. Clearly, however, further research is necessary to establish and explain their limits and accuracy.

6.1.3 One-Dimensional Pressure Spectra

The one-dimensional pressure spectra is defined in the same fashion as the velocity spectra; namely, $F_p(f)$ is the fraction of energy per unit cycle. With the same definitions of the pressure and velocity spectral functions, and the close connection to the velocity field shown in Chapter 2, one would expect many common features between the

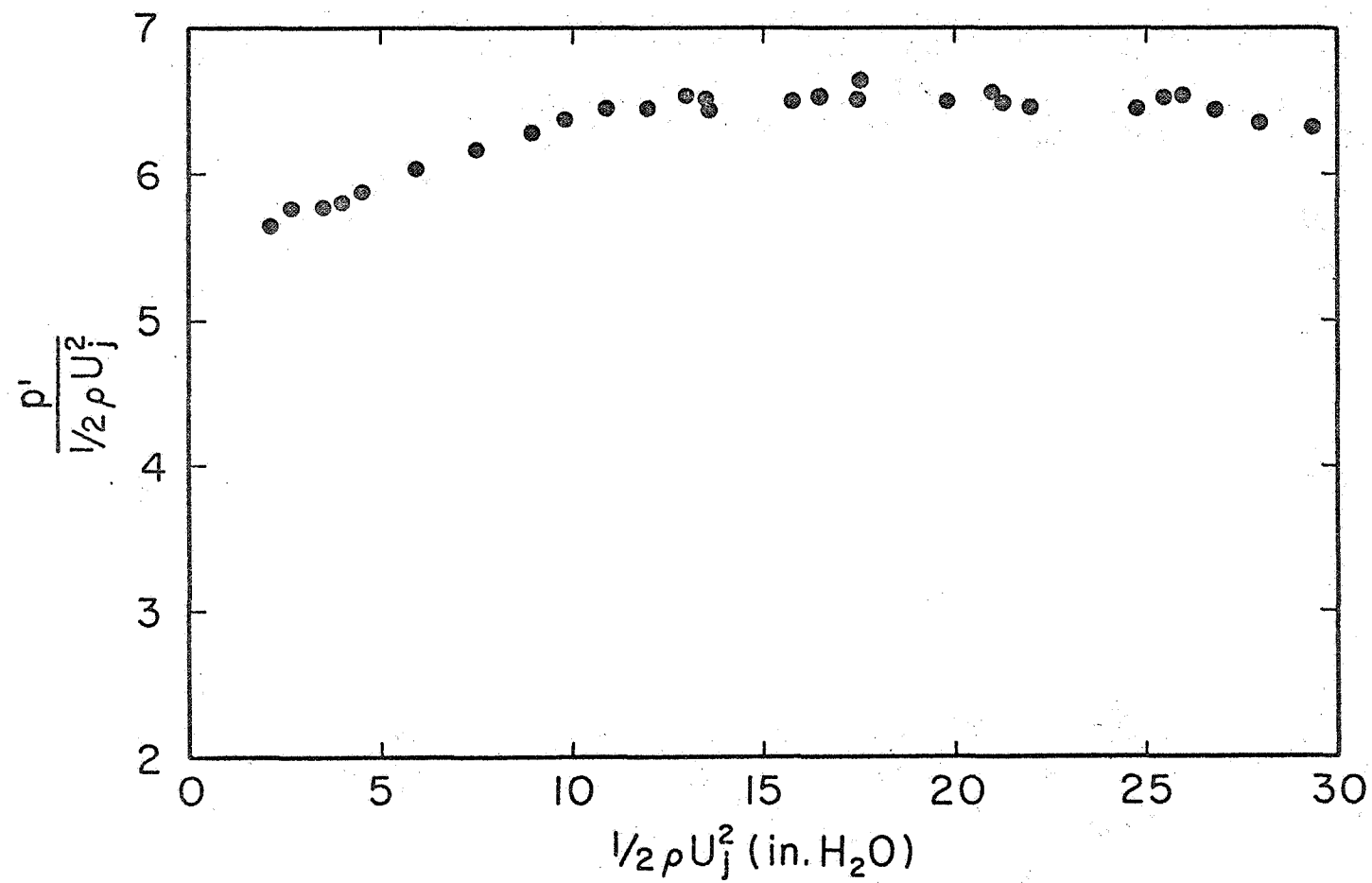


Fig. 6.1-7 Pressure Intensity for Various Jet Speeds. $x/D = 4$, $r/R = 1.0$, 2.5 in. dia. jet.

spectra. Kraichnan's (1956B) work in boundary layers substantiates this general point, however, the theory is insufficiently developed to allow one-to-one comparisons. Figure 6.1-8 shows pressure spectra in terms of the non-dimensional length or wave number based on the frozen pattern hypothesis. This was discussed in Chapter 5. The measurements were taken at the centerline of mixing at several axial distances. The scaling collapse is consistent with the ideas of a nearly frozen spatial structure that increases linearly in size as it is swept downstream at constant convection speed. The peak at $f(x-x_0)/U_j = 2.0$ compares well with the peak in $F_v(f) U_j / x - x_0$ at $f(x-x_0)/U_j = 1.8$. An even better comparison is made in section 6.2.1 where the peak is shown to correspond to a dominant spatial structure in the pressure field. At higher frequencies, or smaller sizes, the spectra passes through a narrow region of $-5/3$ power decay and then enters a region of -3 power decrease. Although the theory isn't sufficiently developed to predict the $-5/3$ decay, similar behavior of the velocity field in the Komogoroff region makes this observation not too surprising. However, the -3 power decrease was somewhat surprising. A similar occurrence was not observed for $F_v(f)$. Since the highest frequency shown is 10 kHz and since the transition from $-5/3$ to -3 seems to scale with size $f(x-x_0)/U_j$ rather than frequency, the -3 power is believed to represent a flow phenomenon.

In summary, the pressure spectra scaling suggests a large slowly changing spatial pattern being convected downstream at a uniform rate. The shape of the spectra compares well with the spectra of transverse velocity except at high frequencies. The structural suggestions are

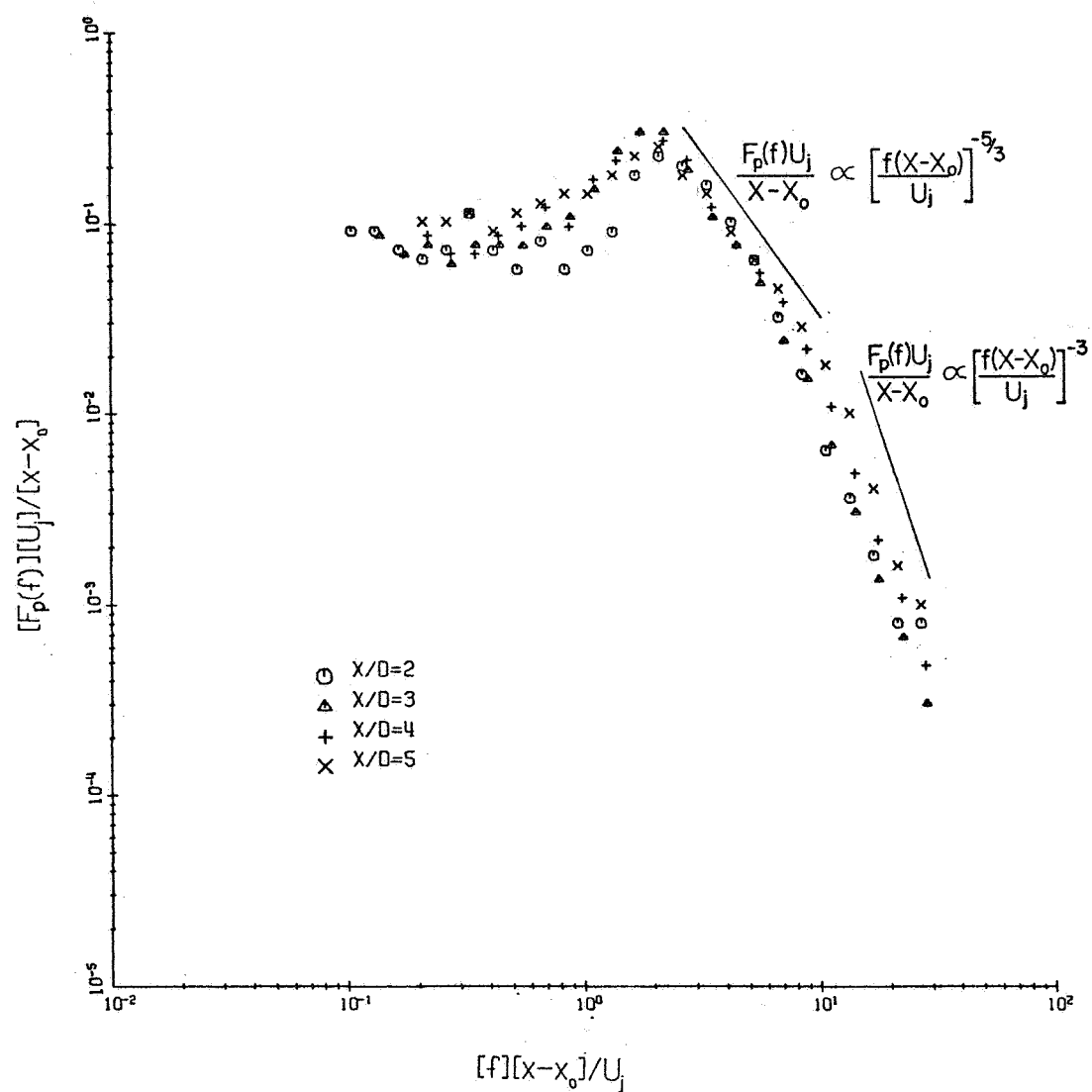


Fig. 6.1-8 Pressure Spectra. $r/R = 1.0$, $U_j = 300$ ft./sec., $x_0 = -2.5$ in.

consistent with space correlations and will be further discussed in section 6.2.1.

6.2 Pressure Two-Point Correlations

As with the velocity correlations, the pressure two-point correlations describe the spatial extent, shape, and time evolution of coherent pressure volumes. The experimental description will hopefully aid in better understanding the role of pressure in mixing layer mechanics. More immediately, the results will be the foundations for a pressure space-time correlation model. With such a model, future research can test Ribner's (1964) dilatation theory of aerodynamic noise generation. If applicable, the far field sound distributions can be computed.

The quantities measured and the terminology used follow the same pattern established for the velocity field in Chapter 5. Briefly, general pressure space-time correlation is defined as

$$R_{pp}(x, r, z, \Delta x, \Delta r, \Delta z, \tau) = \frac{p(x, r, z, t) p(x + \Delta x, r + \Delta r, z + \Delta z, t + \tau)}{(\overline{p^2(x, r, z)} \overline{p^2(x + \Delta x, r + \Delta r, z + \Delta z)})^{1/2}} \quad (6.2-1)$$

As before, the position arguments (x, r, z) and zero separation arguments are suppressed. The space correlations, a special case for zero time delay, are denoted as $R_{pp}(\Delta x)$, $R_{pp}(\Delta r)$, or $R_{pp}(\Delta z)$. The convected frame is that frame for which the time scale is largest. Its autocorrelation is identified as the envelope of the family $R_{pp}(\Delta x, \tau)$ and the convection velocity is determined by the $(\Delta x, \tau)$ coordinates of the envelope.

A separability model of the pressure space-time correlation is introduced briefly here to provide a reference from which to examine the experimental results. The model represents the space-time correlation as a product of functions of one variable.

$$R_{pp}(\Delta x, \Delta y, \Delta z, \tau) = R_{pp}^{(1)}(\Delta x) R_{pp}^{(2)}(\Delta y) R_{pp}^{(3)}(\Delta z) F(\tau - \frac{\Delta x}{U_0}) \quad (6.2-2)$$

The conditions in Eq. 6.2-2 for $\tau = 0$ are referred to as spatial separability which will be shown in section 6.2.3 to be valid. Spatial separability is used to infer zero offset correlations from measurements in which the two sensors are offset. For example, if correlations were made between sensors separated by Δx and offset in the lateral direction by ϵ_z , the zero offset correlation is given by

$$R_{pp}(\Delta x) = \frac{R_{pp}(\Delta x, \epsilon_z)}{R_{pp}(\epsilon_z)} \quad (6.2-3)$$

Assuming the separability hypothesis is valid, the triad of measurements necessary to describe the three-dimensional space-time correlation is a set of correlations for separations along each of the three axes. This triad of measurements was taken at five axial locations as $x/D = 4$. Each triad contained 30 to 50 separations. Several off-axis sets of measurements were taken in the $x-r$ plane to complete the three-dimensional description of the space-time correlation.

6.2.1 Space Correlations

As previously mentioned, the space correlations describe the size and shape of correlated pressure regions. The results which follow will show regions of pressure correlation are larger and significantly

different than regions of correlated turbulent velocity.

The lateral space correlations are important for describing the three-dimensional space-time correlation. In an experimental sense, they are absolutely necessary to use with the separability hypothesis and quantify the effect of the probe lateral offset.

The lateral correlations for five axial positions are shown in Fig. 6.2-1. The correlations were all made with both sensors at the mixing centerline separated along the lateral arc defining the mixing center. For small separations the correlations are exponential,

$$R_{pp}(\Delta z) = \exp(-\Delta z/L_z^p) \quad (6.2-4)$$

The parameter, $L_z^p \sim 0.4$ in., is constant with no measurable axial, hence

does not scale with and the simpler is used in Fig. 6.2-1.

At large distances, the correlations vary as $1/(\Delta z)^m$; where $m \sim 0.5 \pm 0.2$. The uncertainty in m , due to low correlation values, masks variation with axial position. The lateral extent of the exponential region is comparable to the transverse velocity scale and the width of the mixing layer over which the mean velocity gradient is significant. For example, at $x/D = 4$, the exponential region extends to about 0.4 in., the transverse velocity scale is about 0.45 in., and the distance from the mixing centerline to where the mean velocity gradient is 1/2 of its maximum value is about 0.5 inches.

The effect of the lateral offset is quantified by Fig. 6.2-1 as

$R_{pp}(\epsilon_z)$. The offset used for all measurements was $\epsilon_z = .15$ in. corresponding to $\Delta z/D = .06$ --well within the exponential region for all axial positions. The figure also shows the exponential extrapolation

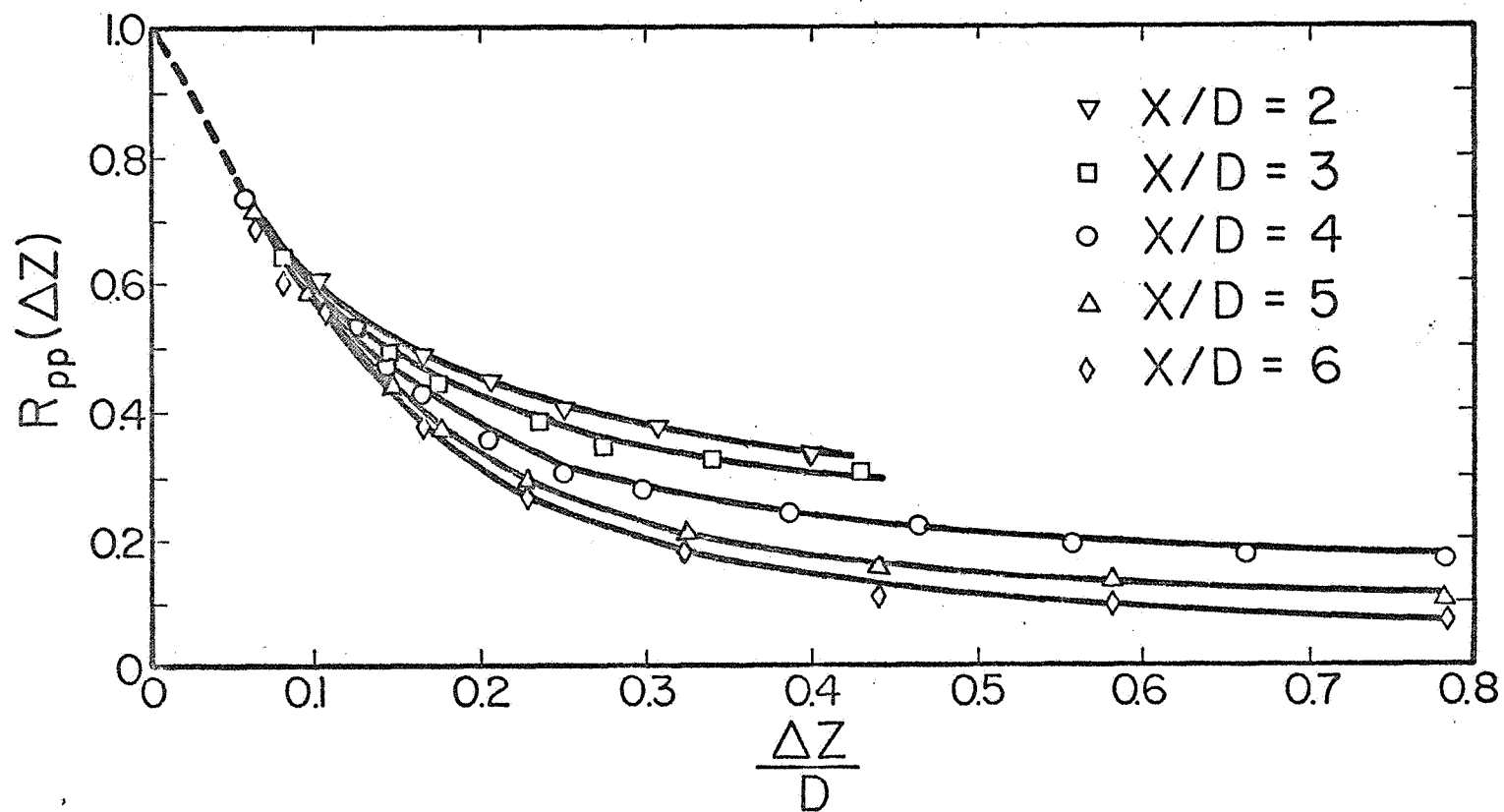


Fig. 6.2-1 Pressure Lateral Space Correlations. $r/R = 1.0$, $D = 2.5$ in., $U_j = 300$ ft./sec.

to $R_{pp}(0) = 1.0$ indicating negligible effect of noise or interference.

The transverse correlations were obtained in the same fashion as the lateral correlations except the probe separation consisted of a constant lateral offset, $\epsilon_z = .15$ in., plus the varying radial separations. The results shown in Fig. 6.2-2 have been corrected for the lateral offset with Eq. 6.2-3. The same general characteristics observed for the lateral correlations are shown in Fig. 6.2-2. At small separations the correlations vary exponentially, at large separations they vary with an inverse power law, and scaling with mixing layer width or axial position is not evident.

At small separations the exponential behavior can be represented by

$$R_{pp}(\Delta r) = \exp(-\Delta r/L_r^p) \quad (6.2-5)$$

As with the lateral correlations, L_r^p shows no axial variation. However, it is significantly larger than L_z^p , ($L_z^p/L_r^p \sim .4$). The extent of the exponential region is about the same as previously observed for lateral correlations and consequently compares well in magnitude to the transverse velocity scale, and region of significant velocity gradient.

At large separations toward the outside of the jet the correlations again obey a power law, $R_{pp}(\Delta r) = \frac{1}{(\Delta r)^n}$. In contrast to the lateral correlations, however, the variation is very slow, $n \sim .05$. For separations toward the jet center, $R_{pp}(\Delta r)$ tends toward a power law dependence then asymptotically becomes flat in the potential core as the jet centerline is approached.

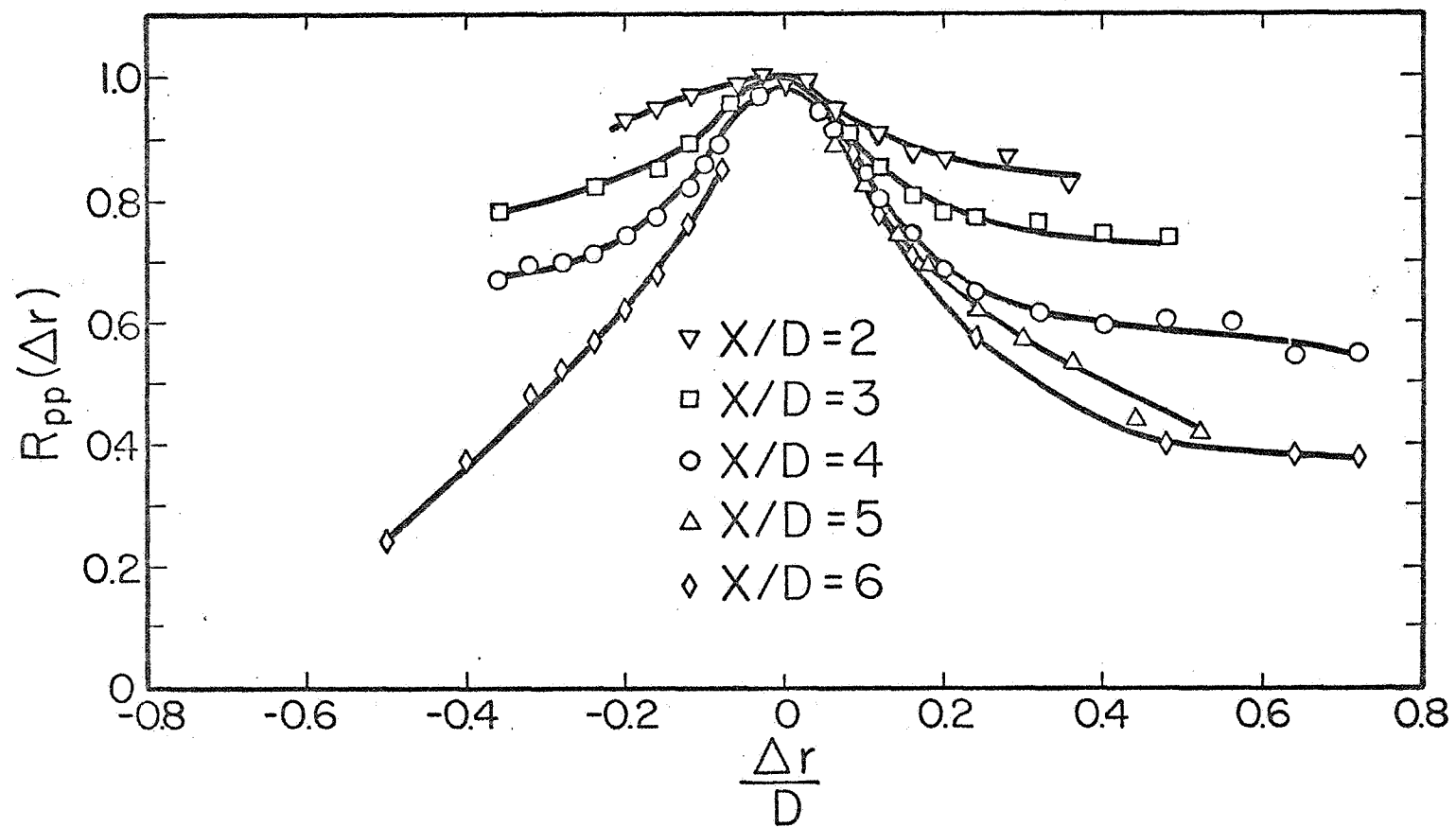


Fig. 6.2-2 Pressure Transverse Space Correlations. $D = 2.5$ in., $U_j = 300$ ft./sec. Stationary Probe at $r/R = 1.0$.

A comprehensive explanation of the lateral and transverse space-time correlations is beyond the scope of the present study. However, a plausible explanation, believed to be worthy of future investigation will be given. The distinctly different exponential and power law regions shown in the results suggest two distinctly different mechanisms. In a paper on noise generation, Ribner (1964) proposed that the total pressure field is composed of two mechanistically distinct components--hydrodynamic pressure due to inertial accelerations and acoustic pressure, a transmission by compressional waves. The exponential region could plausibly be due to the correlation of hydrodynamic pressure responses to an exponentially correlated velocity field. The power law region, which occurs at distances greater than the velocity field correlation length, could be the correlation between the hydrodynamic pressure and its near field acoustic response. This explanation obviously needs much further theoretical and experimental verification. If true, it has important consequences for the Ribner (1964) dilatational theory of aerodynamically-generated sound. This theory uses hydrodynamic pressure correlations as a source term. Consequently, if the explanation is correct, only the exponential region of the radial and transverse correlations should be used as a source term.

The axial space correlations contrast markedly to the transverse and lateral curves. The source of this contrast was pointed out by Kraichnan (1956). He showed that for flows with appreciable mean velocity gradient, the turbulent-mean shear terms require significant negatively-correlated regions in a plane containing the direction of

mean flow. The axial results shown in Fig. 6.2-3 clearly show regions of negative correlation. These results have been compensated for lateral offset with Eq. 6.2-3. Other results show negatively-correlated regions for any correlation containing axial separations.

The correlation curves resemble periodic functions of $\Delta X/(x-x_0)$ damped by ΔX . Other investigators, including Mollo-Christensen (1967) and Mollo-Christensen et al. (1964) have observed curves of this nature for axial pressure correlations of pressure near a turbulent jet. Fuchs (1972) showed qualitative data from a jet mixing region that had this form. The nature of the form suggests representation as

$$R_{pp}(\Delta x) = e^{-\Delta x/L_x^p} \cos \frac{2\pi}{\lambda_c} \frac{\Delta x}{(x-x_0)} \quad (6.2-6)$$

This form, although perhaps not exact, does describe two essential characteristics or length scales of the correlations which is the present objective.

Two length scales are evident in the results, and with the aid of Eq. 6.2-6, are identified as a wavelength of periodicity λ and a damping described by L_x^p . The wavelength of the structure grows linearly with downstream position. From measurements of the first zero crossing, which corresponds to a quarter wavelength, it was found that $\lambda \approx 0.3(x-x_0)$ so that the scale constant in Eq. 6.2-6 is $\lambda_c = 0.3$. The virtual origin ($x_0 = -2.5$ in.) necessary to scale the data is much larger than was found for velocity ($x_0 = -0.4$ in.). The wavelength can be roughly related to the peaks in the pressure spectrum by considering the spatial structure to be a fixed pattern swept by a

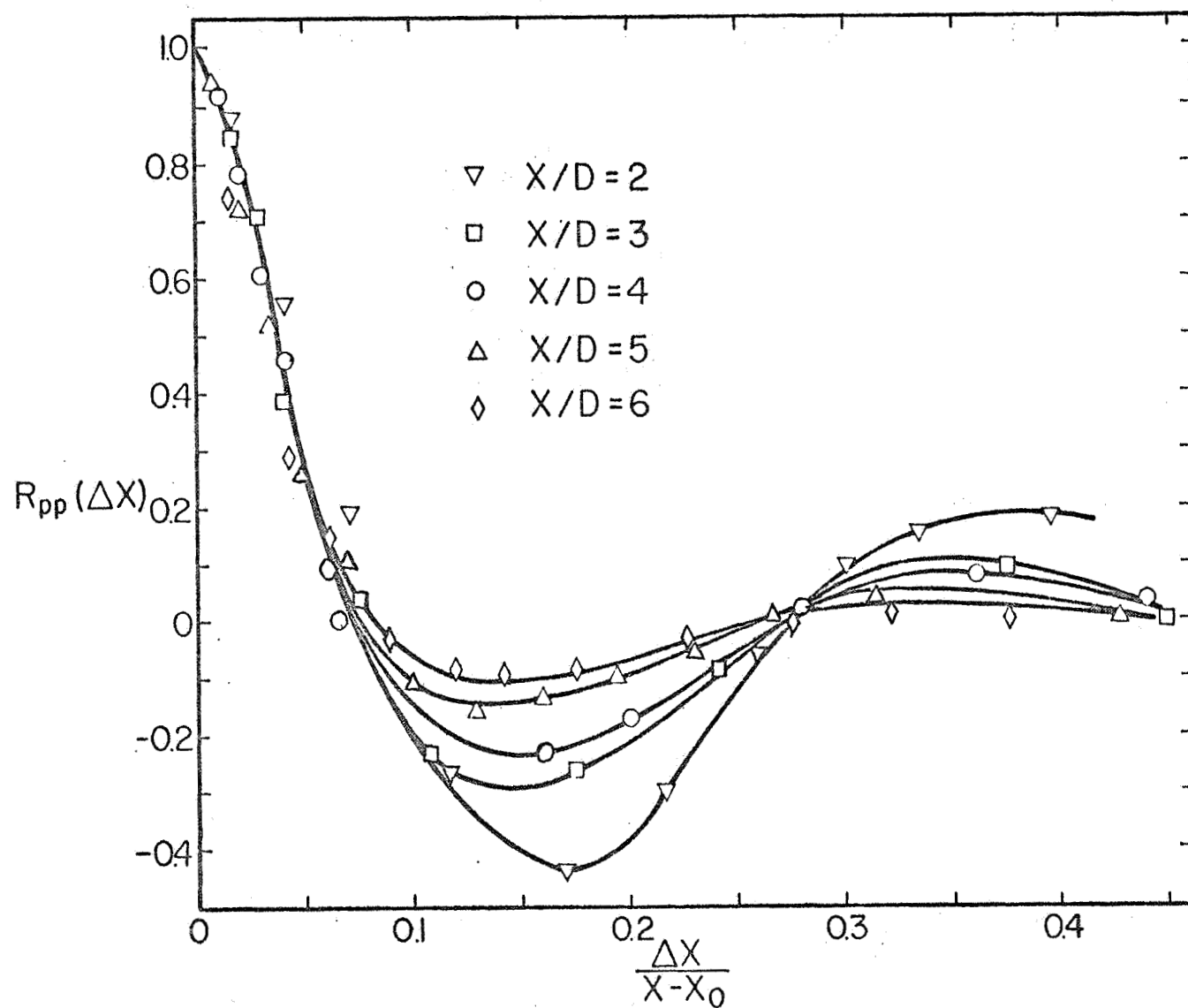


Fig. 6.2-3 Pressure Axial Space Correlations. $r/R = 1.0$, $D = 2.5$ in., $U_j = 300$ ft./sec.

measuring point. The relation is

$$f_{PK} = U_c / \lambda \quad (6.2-7)$$

so that $f_{PK}(x-x_0)/U_c = 3.3$ which agrees well with the spectrum peak at $f_{PK}(x-x_0)/U_c = 3.4$.

The damping of the space correlations is also shown in Fig. 6.2-3. The damping obviously is not solely a function of $\Delta x/(x-x_0)$. In fact, by evaluating the variations of R_{pp} at the first minimum $\Delta x/(x-x_0) = .15$, the damping was found to be a strong function only of the separations. Thus L_x^p in Eq. 6.2-4 varies little with axial position and can be considered a constant, $L_x^p \approx 1.5$ in.

In summary, the spatial correlations, for the separations in lateral and transverse directions, are quite different than those for separations in the flow direction. This was predicted by Kraichnan (1956). The lateral and transverse correlations are characterized for small separations by an exponential decrease. At large separations the correlations vary according to an inverse power law. The size of the exponential region increases with mixing layer width and its size is comparable to the turbulent velocity integral scale.

The axial correlations are characterized by an exponentially-damped cosine. The wavelength is larger than, but increases in proportion to the mixing layer width. The degree of damping however varies slowly.

6.2.2 Space-Time Correlations--The Convected Frame

As discussed earlier, the axial space-time correlations give evidence of the time evolution and speed of the convected frame. These

phenomena can be observed in Fig. 6.2-4. Each family of curves results from two sensors located on the mixing center. One is fixed and the other is separated by increasing downstream distances. The three families are results from three different fixed sensor positions. The curves are normalized according to Eq. 3.2-3 to compensate for lateral offset. With the figures, one can envision the spatial structure in Fig. 3.2-3 becoming less correlated as it is swept downstream. Two important parameters, the decorrelation time and convection velocity, are traditionally used to describe this phenomenon. The convection velocity is defined by the time and space coordinates of the space-time envelope and was computed for the results in Fig. 6.2-4 as well as many other families not shown. Variation of convection velocity with separation, if any, was less than 5% and therefore not measurable. This indicates that convection velocity variations with structure size, as discussed for velocity by Wills (1964), is either negligibly small or that the flow is so dominated by a single size structure that velocities of other size structures are not discernible. The damping of the axial space correlation, which has been discussed, and the lack of time separability, which will be discussed, suggests the possibility of dispersion. This dispersion, as explained by Wills, could be the result of different-sized structures convecting at different speeds. A tentative conclusion is that convection velocity is wave number dependent, but that the flow is so dominated by a single size structure that the effects are small.

The convection velocity results are shown in Fig. 6.2-5 for several points on the centerline of mixing. Figure 6.2-6 shows convection

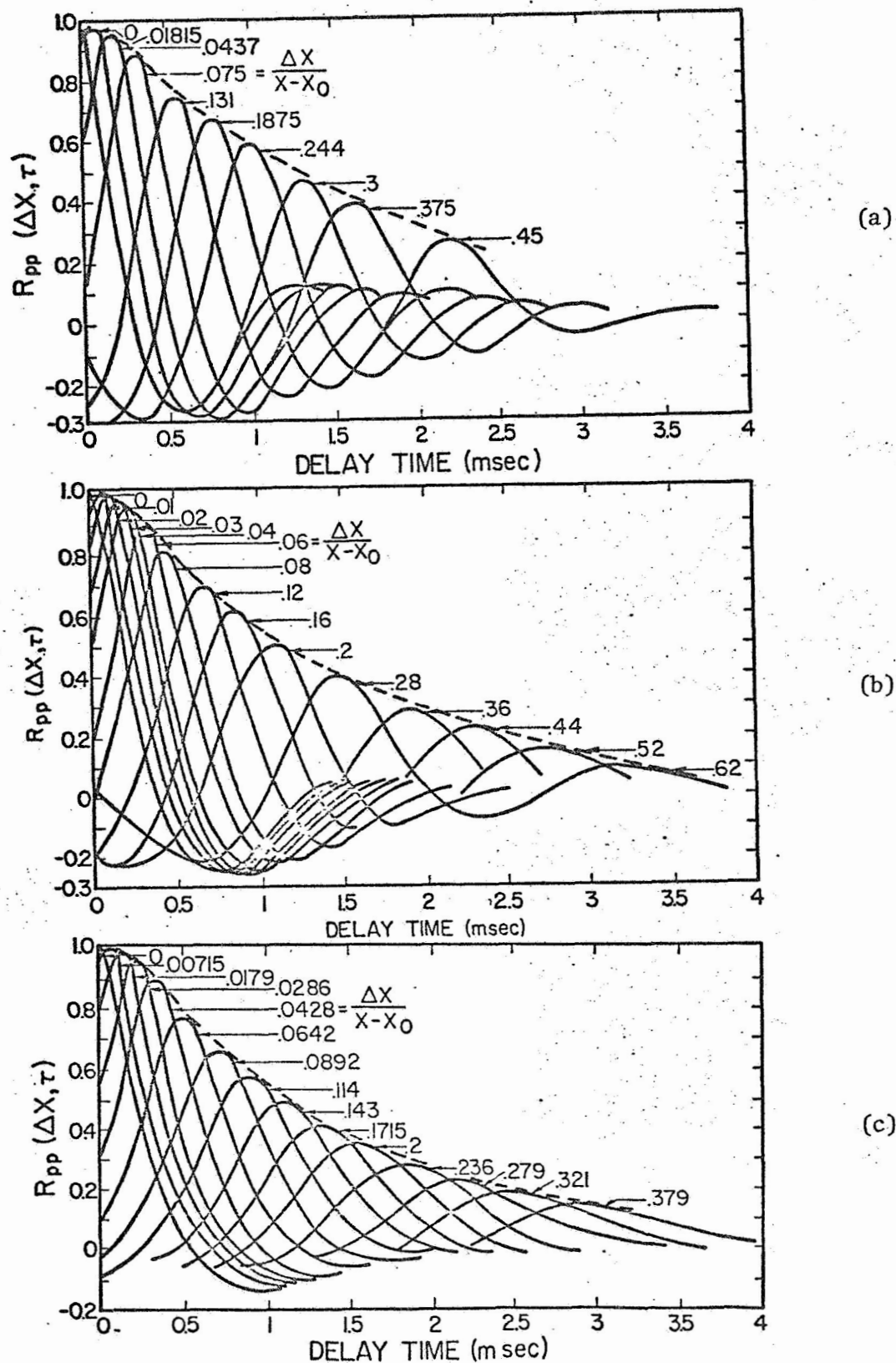


Fig. 6.2-4 Pressure Space-Time Correlations. $r/R = 1.0$, $U = 300$ ft./sec., $x = -0.4$ in.; (a) $x/D = 3$; (b) $x/D = 4$; (c) $x/D = 6$

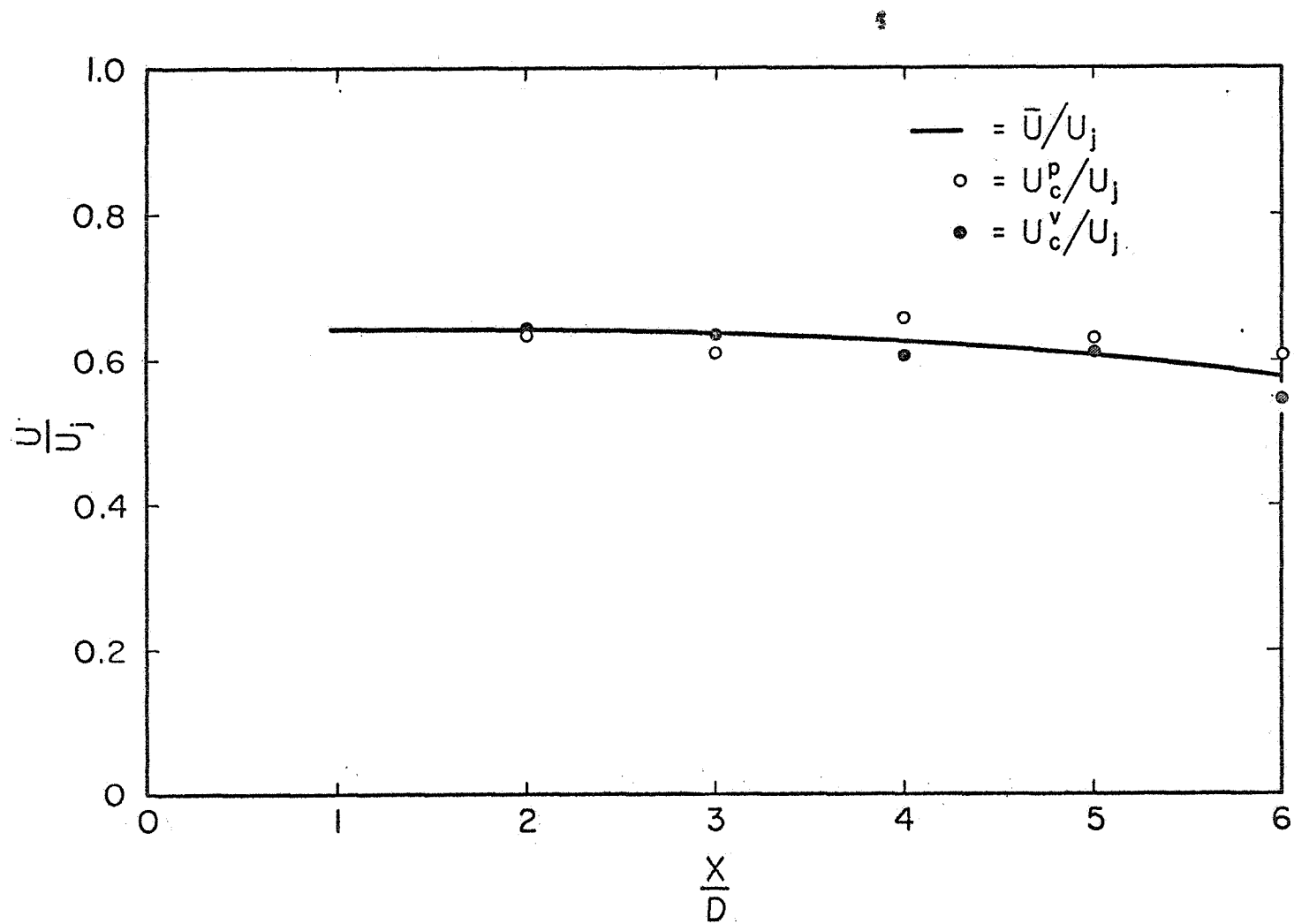


Fig. 6.2-5 Pressure Convection Velocity. $r/R = 1.0$. $U_j = 300$ ft./sec., $D = 2.5$ in.

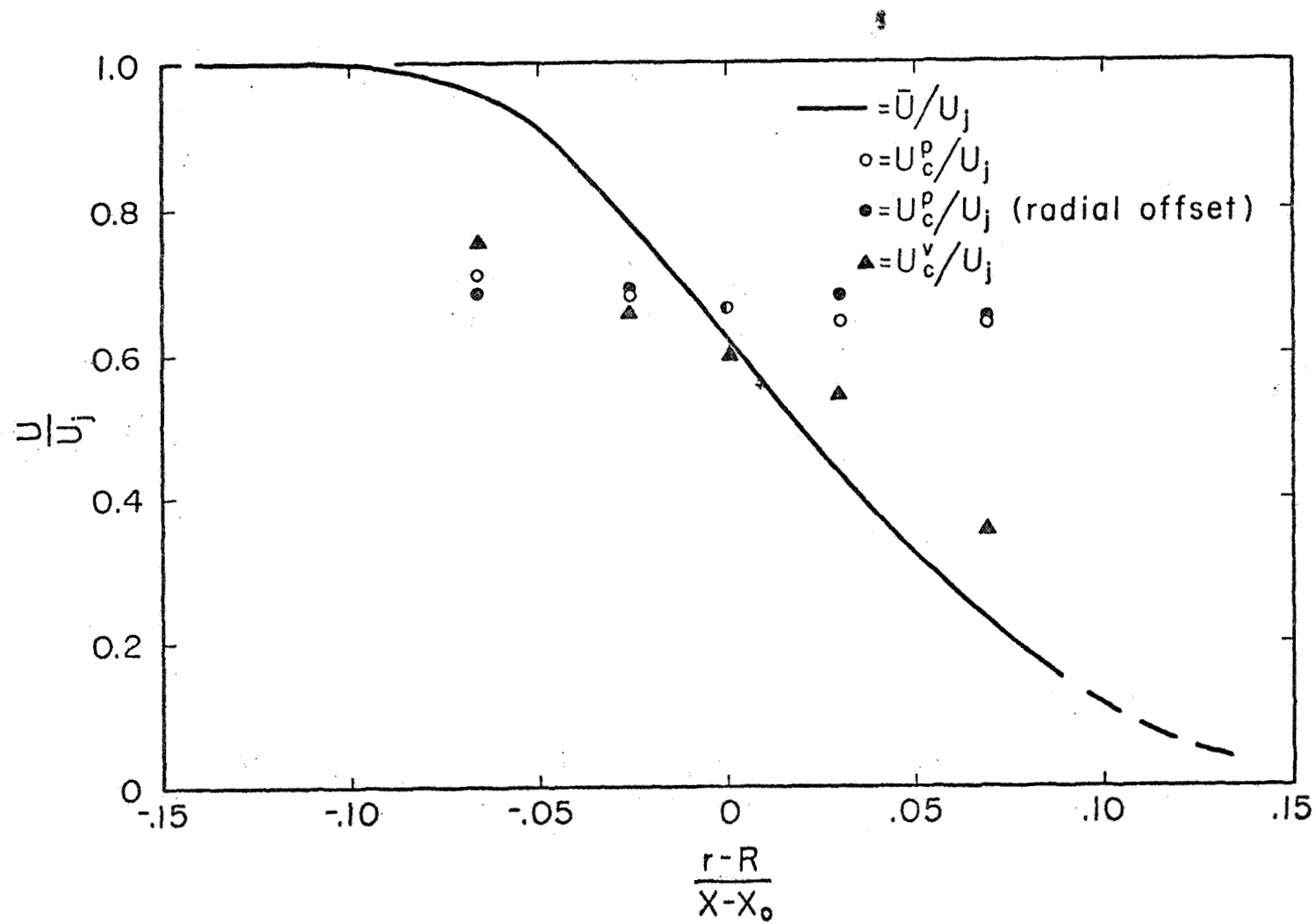


Fig. 6.2-6 Pressure Convection Velocity $x/D = 4$. $U_j = 300$ ft./sec., $R = 1.25$ in.

velocities for correlations with radial offsets $R_{pp}(\Delta x, \Delta r)$. The offset Δr was held constant for a series of downstream separations and corresponds to $r-R$ in the figure. The most significant point about the convection velocity is that it is almost constant throughout the mixing layer. The same constancy was also verified for a range of jet velocities from 100 to 300 fps. The lack of transverse variation is consistent with our earlier observation that pressure was correlated over large transverse regions. The idea of large correlated regions convecting downstream at a uniform speed is thus solidified.

The time evolution of coherence in the convected frame is indicated by the envelope of the space-time correlations. This envelope, denoted $R_{pp}^L(\tau)$, is routinely interpreted as the auto-correlation of the convected frame. Figure 6.2-4 shows a slow variation in $R_{pp}^L(\tau)$ for small separations giving way to an exponential decay beyond about 1/2 inch.

The whole evolution can be characterized, except for small separations, by the integral time scale T_L^P which is defined by

$$T_L^P = \int_0^{\infty} d\tau R_{pp}^L(\tau) \quad (6.2-6)$$

Since $R_{pp}^L(\tau)$ was found to be exponential, except for small separations, T_L^P could be inferred from the data with a least squares fit to $R_{pp}^L(\tau) = e^{-\frac{\tau}{T_L^P}}$. The results from this fit were compared to several integrated curves and found to differ less than 6%. The non-exponential region near the origin obviously contributes to the difference.

However, the exponential fit does allow evaluation of τ_L^P when data for large separations is not available, and when examining the envelope of correlations with radial offsets. Considering these advantages outweigh the disadvantages, τ_L^P was consistently evaluated from the least squares fit.

The axial variation of the time scale at the mixing centerline is shown in Fig. 6.2-7. The time scale of the transverse turbulence is also shown. The figure shows that τ_L^P scales vary poorly with axial position. The same trend is also observed for the transverse turbulence. Another important feature is that τ_L^P is almost twice as large as τ_L^V . The time scale for pressure is nearly constant across the mixing layer as shown in Fig. 6.2-8. Also constant as shown are the time scales from the envelope of $R_{pp}(\Delta x, \Delta r)$. The radial offset is plotted on the figure as $r-R$.

In summary, the results confirm the concept described in section 6.2.1 of a large, growing structure being swept downstream at a rate that is uniform across the mixing layer. The structure exponentially decorrelates with time or equivalent path length; however, the time or length scale for decorrelation is large.

6.2.3 Space-Time Correlation Separability

To be of most engineering use, the space-time data should be modeled. Additionally, a unified model helps structure the experimental data so the physical phenomena become clearer. The present purpose is to model the general phenomena and provide a structure for future detailed work.

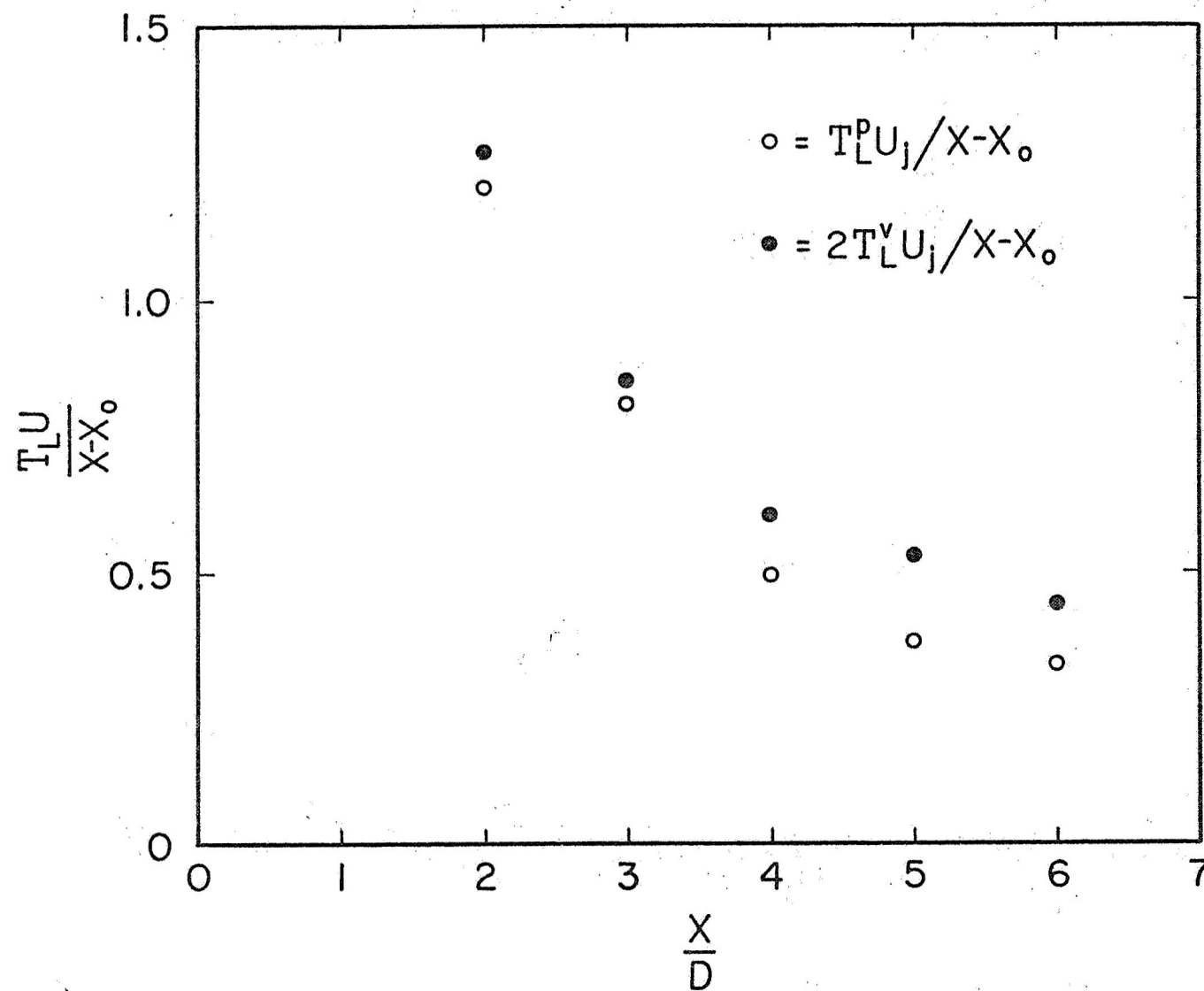


Fig. 6.2-7 Pressure Convected Frame Time Scale, $r/R = 1.0$. $U_j = 300$ ft./sec.

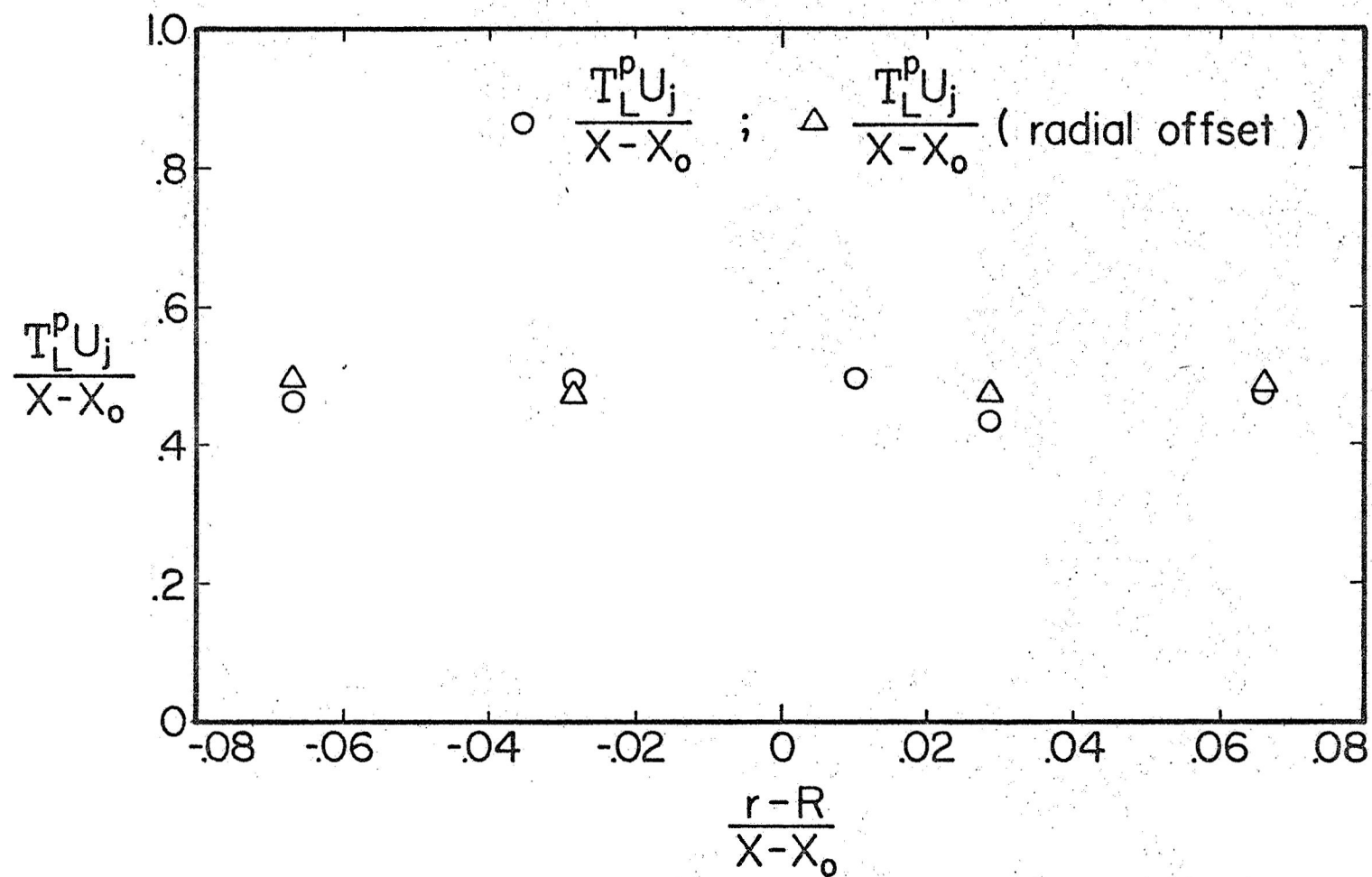


Fig. 6.2-8 Pressure Convected Frame Time Scale, $x/D = 4$. $U_j = 300$ ft./sec.

Mollo-Christensen's (1967) model of sound emitters and the Lau et al. (1972) pressure-related mixing model suggest a separability hypothesis. The hypothesis simply states that the four-dimensional space-time correlation can be expressed as a product of functions of independent arguments.

$$R_{pp}(\Delta x, \Delta y, \Delta z, \tau) = R_{pp}^{(1)}(\Delta x) R_{pp}^{(2)}(\Delta r) R_{pp}^{(3)}(\Delta z) F(\tau - \frac{\Delta x}{u_c}) \quad (6.2-7)$$

Some general characteristics of the product functions are immediately obvious. All must be unity for zero arguments. Space correlations can be identified with the functions.

$$\begin{aligned} R_{pp}(\Delta x) &= R_{pp}^{(1)}(\Delta x) F(-\Delta x/u_c) \\ R_{pp}(\Delta r) &= R_{pp}^{(2)}(\Delta r) \\ R_{pp}(\Delta z) &= R_{pp}^{(3)}(\Delta z) \end{aligned} \quad (6.2-8)$$

The auto-correlation is identified with F and shows it must also be an even function

$$R_{pp}(\tau) = F(\tau) \quad (6.2-9)$$

A transformation into the convected frame identifies $R_{pp}'(\Delta x)$. The transformation is

$$\begin{aligned} \Delta \tilde{x} &= \Delta x - \tau u_c \\ \Delta \tilde{r} &= \Delta r \\ \Delta \tilde{z} &= \Delta z \end{aligned} \quad (6.2-10)$$

The result of the transformation is

$$R_{pp}(\Delta\tilde{x}+Uc\tau, \Delta\tilde{r}, \Delta\tilde{z}, \tau) = R_{pp}^{(1)}(\Delta\tilde{x}+Uc\tau) R_{pp}^{(2)}(\Delta\tilde{r}) R_{pp}^{(3)}(\Delta\tilde{z}) X_{(6.2-11)} \\ \times F\left(-\frac{\Delta\tilde{x}}{Uc}\right)$$

If $\Delta\tilde{x} = 0$, then $(Uc\tau, \tau)$ is the locus of points defining the envelope of the space-time correlation. At $\Delta\tilde{r} = \Delta\tilde{z} = 0$, this envelope is the auto-correlation in the convected frame previously discussed. Thus the $R_{pp}^{(1)}$ function is identified

$$R_{pp}^L(Uc\tau) = R_{pp}^{(1)}(Uc\tau) \quad (6.2-12)$$

The hypothesis can be checked for consistency through its many implications for the measurements. A natural division seems to investigate spatial separability (for $\tau = 0$) and then to check for model consistency in predicting both space and time variations. As discussed, spatial separability for lateral separations was established to compensate for lateral sensor offset. However, a more stringent test is for separability across the shear layer. Figure 6.2-9 shows the very encouraging results of such a test. $R_{pp}(\Delta x, \Delta r, \epsilon_z)$ was measured along a line 30° from the axial direction. The separability hypothesis in Eq. 6.2-7 implies this measurement can be predicted from the results of a radial transverse $R_{pp}(\Delta r, \epsilon_z)$ and an axial transverse $R_{pp}(\Delta x, \epsilon_z)$ and the lateral transverse $R_{pp}(\epsilon_z)$ to correct for offset. The results show the implications of the hypothesis are correct.

Of the many implications of space and time separability, one of the more interesting is a generalization of Taylor's frozen pattern hypothesis. Equation 6.2-7, Eq. 6.2-9 and Eq. 6.2-12 indicate that

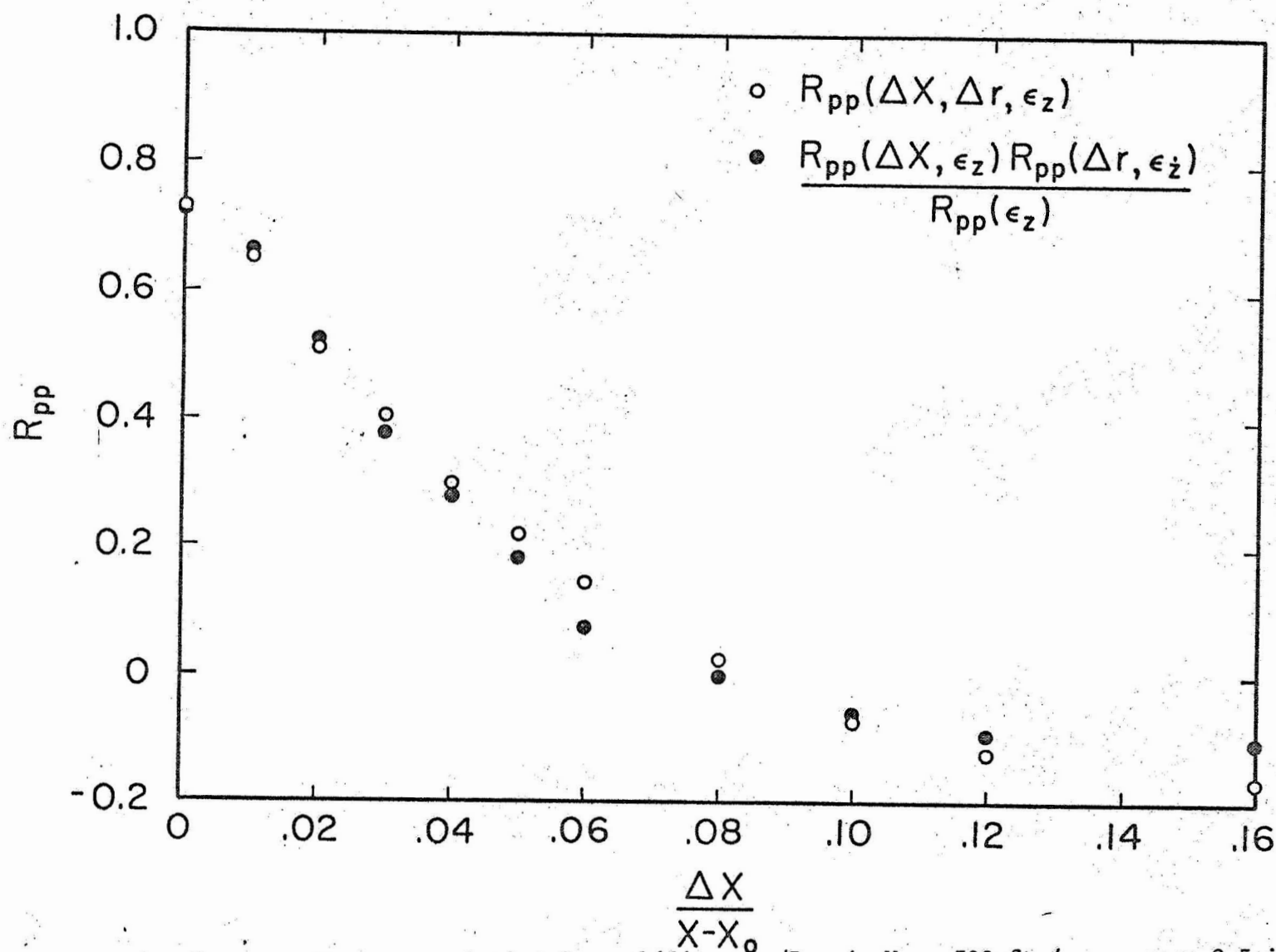


Fig. 6.2-9 Pressure Space Correlation Separability. $x/D = 4$, $U = 300$ ft./sec., $x = 2.5$ in., Stationary Probe at $r/R = 1$.

the space-time correlation should be predicted from the auto-correlations with the amplitude fixed by the convected frame envelope. For axial separations

$$R_{pp}(\Delta x, \epsilon_z, \tau) = R_{pp}^L(Uc\tau, \epsilon_z) R_{pp}(\tau - \frac{\Delta x}{Uc}) \quad (6.2-13)$$

Fig. 6.2-10 shows measured $R_{pp}(\Delta x, \epsilon_z, \tau)$ and the value predicted with Eq. 6.2-13 from measurements of $R_{pp}^L(Uc\tau, \epsilon_z)$ and $R_{pp}(\tau - \Delta x/Uc)$. The predicted amplitudes are correct, however, the model does not account for a dispersive effect that broadens and skews the space-time correlation.

Another case of space-time separability that is useful is prediction of the three-dimensional envelope of the space-time correlation. Using the same reasoning as before, Eq. 6.2-7 implies

$$R_{pp}(Uc\tau, \Delta r, \tau) = \frac{R_{pp}^L(Uc\tau, \epsilon_z, \tau) R_{pp}(\Delta r, \epsilon_z)}{(R_{pp}(\epsilon_z))^2} \quad (6.2-14)$$

Using Eq. 6.2-14 along with the measured envelope, $R_{pp}^L(Uc\tau, \epsilon_z, \tau)$ at $x/D = 4$ on the centerline of mixing, the transverse space correlation $R_{pp}(\Delta r, \epsilon_z)$, and the lateral correlation $R_{pp}(\epsilon_z)$ were used to compute the envelope of curves with a radial offset $R_{pp}^L(Uc\tau, \Delta r, \tau)$. Figure 6.2-11 shows the agreement is good.

In conclusion, the separability hypothesis allows prediction of the four-dimensional space-time correlation in terms of four independent measurements of axial, radial, lateral, and time variation. Spatial separability seems to be valid. Time separability; however, is only

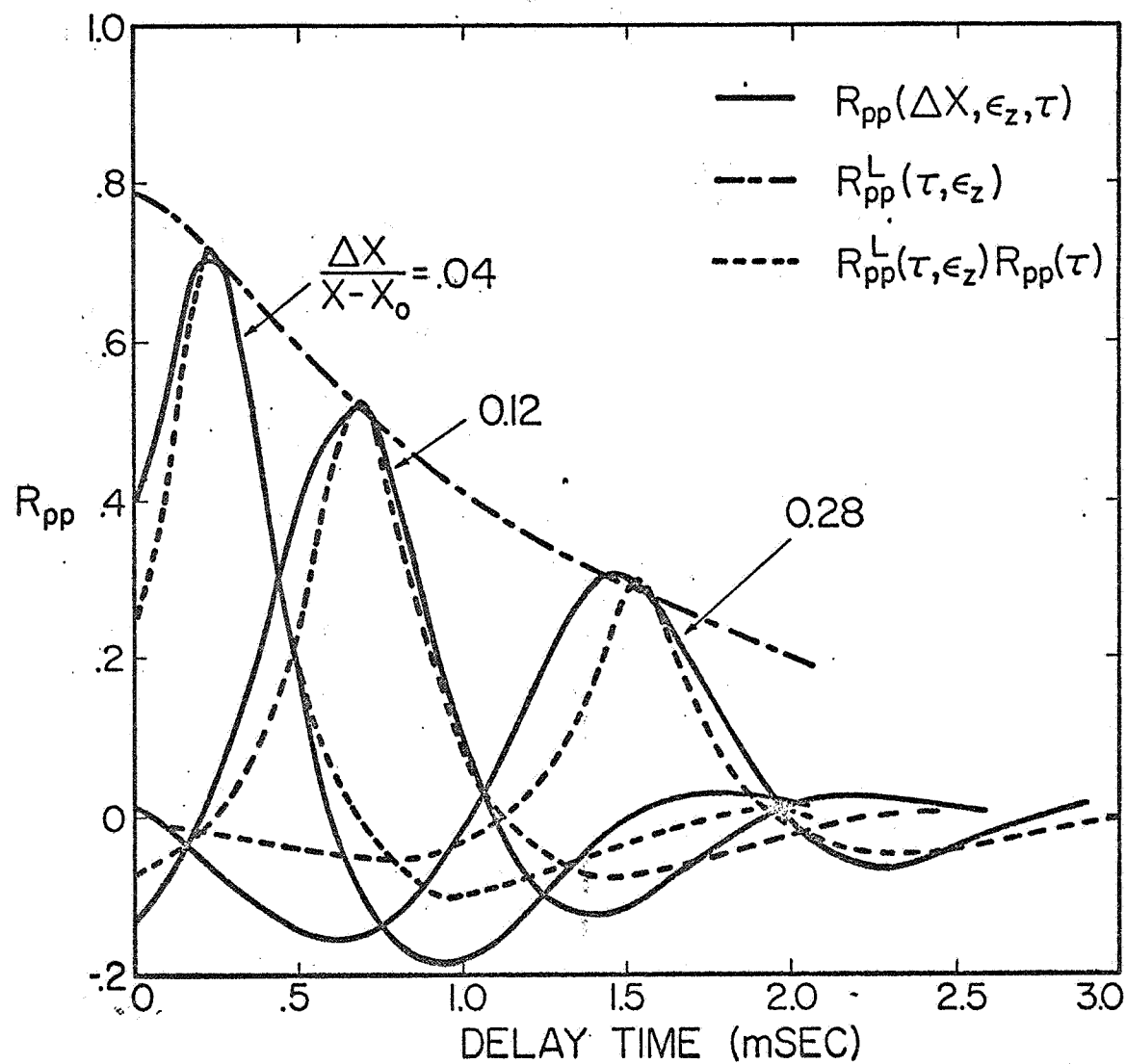


Fig. 6.2-10 Pressure Space-Time Correlation Separability. $x/D = 4$, $r/R = 1.0$, $U_j = 300$ ft./sec.

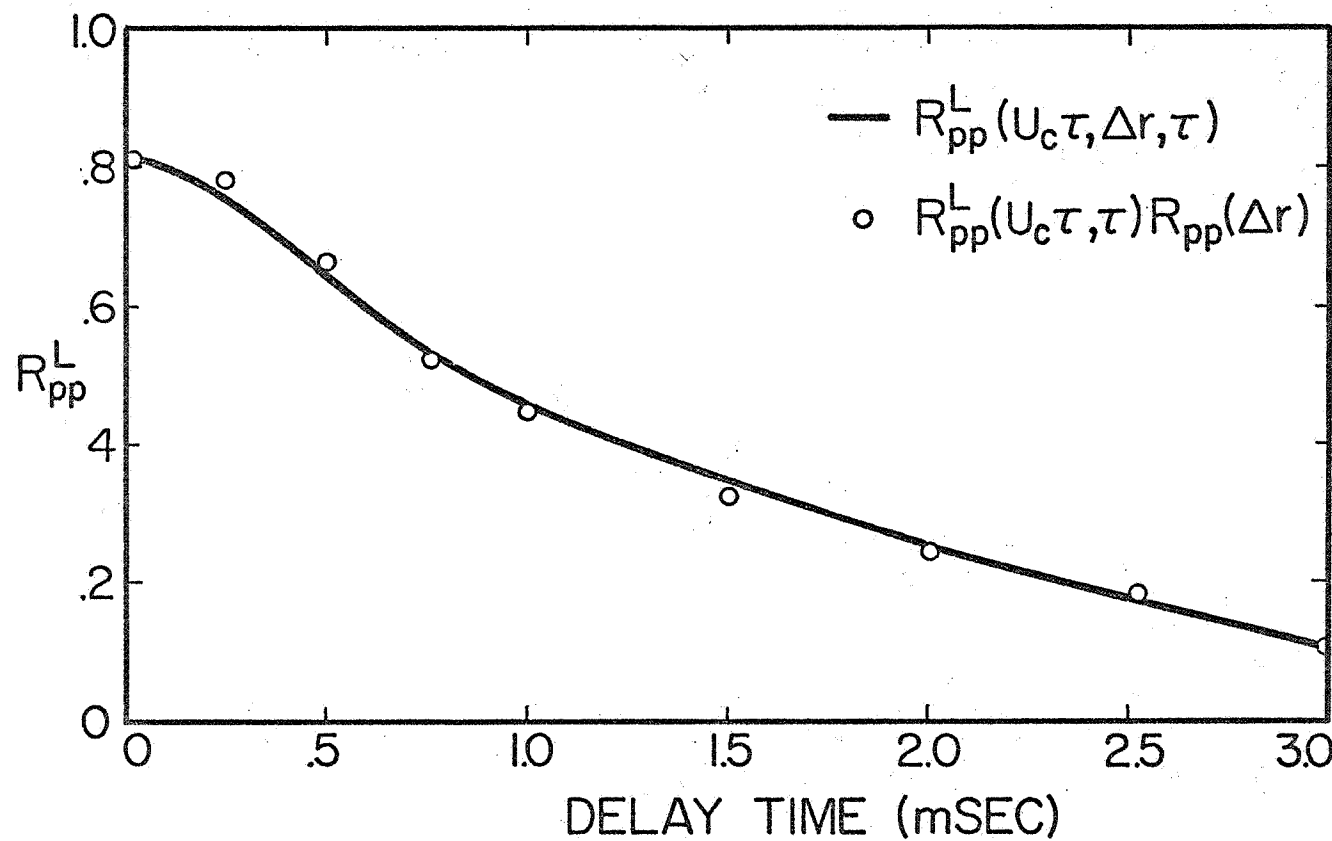


Fig. 6.2-11 Pressure Space-Time Envelope Separability. $U_j = 300$ ft./sec.; Stationary Probe at $x/D = 4$, $r/R = 1$.

approximate and requires further details in the model to account for what appears to be time dispersion of the correlated volume.

7. SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS FOR FUTURE RESEARCH

The research conducted involved three distinct but related parts. First, the theory established the relation between fluctuating static pressure and fluctuating velocities in a turbulent jet mixing region. Second, an instrument was refined, tested, and shown appropriate for pressure measurement in the jet turbulent shear layer. Third, measurements of velocity and pressure fields were made and interpreted.

The theory in Chapter 2 showed that the static pressure in an incompressible mixing layer could be divided into a turbulent-turbulent component and a turbulent-mean shear component. It was clearly shown that the turbulent-mean shear pressure correlations depend upon transverse velocity intensity, mean velocity gradient, and turbulent velocity scale. It was found that the turbulent-mean shear intensity varies linearly with these quantities. The experimental results are consistent with Kraichnan's (1956, 1956B) idea that the turbulent-mean shear term dominates; however, future analytical effort, to more accurately deduce the turbulent-turbulent term, is needed to solidify this idea. Another important area for future theoretical research would be to generalize the results for turbulent-mean shear intensity to include turbulent-mean shear space-time correlations. A relaxation of the local homogeneity assumption would also be helpful. Such investigation could further explain the nature of the pressure spatial structure.

The most important conclusion from the pressure transducer development and tests was that the instrument can accurately measure fluctuating static pressure in the jet mixing layer. The analysis, in Chapter 3, showed the limiting error in pressure measurement was from fluctuating velocity. The axial velocity error was eliminated by probe design reducing the axial pressure coefficient to zero. The error from normal fluctuating velocities could not be eliminated; however, the analysis shows the errors are much smaller than the fluctuating pressure in the jet mixing layer--rms pressure errors were less than 5%. The model of the pressure transducer errors and the techniques presented form a basis for evaluating the pressure transducer's ability to make measurements in other flows or to evaluate errors in measuring other quantities such as pressure-velocity correlations.

The velocity field measurements show approximate agreement with well established scaling laws for a jet mixing region. These are that mean velocity, fluctuating velocities, and shear stresses are self similar, and that fluctuating velocity scales increase linearly with downstream distance. It was found that the deviation from self similarity by both fluctuating velocity and shear stress is due to gradual changes in fluctuating velocity magnitude and is not due to changes in the turbulent structure. In a region near the mixing centerline, turbulent velocity scales, structure, convection velocity, and convected frame time scales were found to vary slowly both in the transverse and axial directions. This is consistent with the assumption of local homogeneity made in the pressure theory development. However, the more rapid variation in the transverse direction of the turbulent

intensity and mean velocity gradient weakens the assumption. On the mixing centerline this produces higher calculated pressure intensities than were measured.

Kraichnan's (1956) elongated isotropic model for turbulent velocity correlations was not found to be accurate in its full generality. It fails when more than one velocity component is considered. This precludes reliable calculations of pressure from turbulent-turbulent interactions. However, parts of the model were found to be accurate, in an empirical sense, and were used to represent the transverse velocity correlation for turbulent-mean shear pressure intensity calculations.

Variation of measured pressure near the mixing centerline was found to show good agreement with the theoretical predictions for the turbulent-mean shear pressure. The rms pressures are about 17% higher than predicted by Eq. 6-1 because of the weakness of the local homogeneity assumption. However, the dependence of pressure upon mean velocity gradient, transverse fluctuating velocity, and scale size as shown in Eq. 6-1 is closely followed. Scaling laws based upon Eq. 6-1 were shown to be valid for jet size and speed in a limited number of tests. Further tests are needed to establish the effects of compressibility and velocity ratio.

Measurements of spectra, space correlations, and space-time correlations consistently suggest a large, slowly varying pressure structure uniformly convecting downstream. Within a turbulent velocity scale length of the mixing centerline, where significant mean velocity gradient and turbulent intensity exists, the lateral and transverse

space correlations were found to decrease exponentially. Axially, the pressure space correlation has a damped periodic structure. The negative regions of this periodic structure were identified by Kraichnan (1956) as distinct features of the turbulent-mean shear interaction. The experimental results show the wavelength of the periodicity, which increases linearly with downstream distance, produces a pronounced spike in the pressure and transverse velocity spectra measured in the mixing layer and the axial fluctuating velocity spectra measured in the potential core. The exponential length scale of the lateral and transverse correlations as well as exponential damping of the axial periodic structure was determined to be constant. At large separations the lateral and transverse correlations undergo a power law decay. The power law decay, although not fully understood, may be a characteristic of the acoustic field of the mixing layer and certainly merits further investigation.

The convection velocity of the pressure structure is almost constant across the mixing layer. The evolution of the structure in the convected frame was shown to be very slow. It evolves at about one half the rate of the transverse velocity structure.

The measurements determined that the three-dimensional pressure space correlation is separable, that is, it can be expressed as the product of correlations in the three orthogonal directions. An extension including space-time separability is not exact. Apparently, it fails to account for time dispersion of the correlated volume and requires further investigation.

Applications of these results to the aerodynamic noise problem through the Ribner (1964) model seem to be a very promising area for future research. The uniformities of pressure scales and convection velocities plus the exactness and simplicity of spatial separability suggests the almost ideal possibility of accurate yet simple models for aerodynamic noise generation.

APPENDIX I TURBULENCE-MEAN SHEAR PRESSURE INTENSITIES IN AXISYMMETRIC TURBULENCE

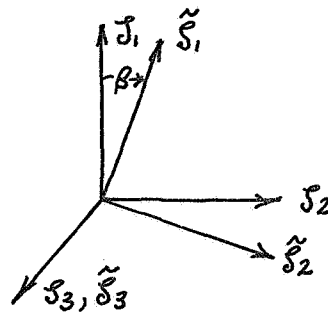
The incentives, for using an axisymmetric model stem from its logical completeness and generality compared to an empirical model. However, equally important, Weber (1973) has shown the axisymmetric forms to be a good representation of mixing layer turbulence.

As developed in Chapter 2, the governing relation between and the velocity field is

$$P_{TM}^2 = \frac{1}{2\pi} \left(\frac{\partial \bar{U}}{\partial y} \right)^2 \rho^2 \int_{\vec{S}} d\vec{S}^3 \bar{U}_2 \bar{U}_2 (\vec{S}) \left(1 - \frac{\vec{S}^2}{|\vec{S}|^2} \right) \frac{1}{|\vec{S}|} \quad (\text{A-1})$$

An incompressible Stokesian fluid with local homogeneity and constant mean velocity gradient have been assumed.

Axisymmetric turbulence is now assumed. Axisymmetry is used in the sense defined by Batchelor (1946) in that the turbulent correlation tensors are invariant to rotations about a principal direction $\vec{S}_1$ and invariant to reflections through a plane perpendicular to $\vec{S}_1$. The axisymmetric system is taken as the result of a rotation in the mean flow-mean gradient plane.



The transformation equations are:

$$\tilde{S}_1 = S_1 \cos \beta + S_2 \sin \beta \quad (\text{A-2a})$$

$$\tilde{S}_2 = -S_1 \sin \beta + S_2 \cos \beta \quad (\text{A-2b})$$

The velocity space correlations transform by

$$\overline{u_2 u_2} = \sin^2 \beta \overline{\tilde{u}_1 \tilde{u}_1} - 2 \sin \beta \cos \beta \overline{\tilde{u}_1 \tilde{u}_2} + \cos^2 \beta \overline{\tilde{u}_2 \tilde{u}_2} \quad (\text{A-2c})$$

The pressure equation (Eq. A-1) is then transformed with Eq. A-2.

$$P_{TM}^2 = \frac{\rho_0}{2\pi} \left(\frac{\partial \tilde{u}_1}{\partial y_2} \right)^2 \int_{\tilde{S}} d\tilde{S}^3 \left[\frac{1}{\tilde{S}} - \frac{\cos^2 \beta \tilde{S}_1^2 - 2 \sin \beta \cos \beta \tilde{S}_1 \tilde{S}_2 + \sin^2 \beta \tilde{S}_2^2}{\tilde{S}^3} \right] \times$$

$$\times \left[\sin^2 \beta \overline{\tilde{u}_2 \tilde{u}_2} + 2 \sin \beta \cos \beta \overline{\tilde{u}_1 \tilde{u}_2} + \cos^2 \beta \overline{\tilde{u}_1 \tilde{u}_1} \right] \quad (\text{A-3})$$

The reflection property of axisymmetry implies that $\overline{\tilde{u}_1 \tilde{u}_1}(S)$ and $\overline{u_2 u_2}(S)$ are even functions of $\tilde{S}_1$, but that $\overline{\tilde{u}_1 \tilde{u}_2}(S)$ is an odd function of $\tilde{S}_1$. Thus several terms above integrate to zero leaving

$$P_{TM}^2 = \frac{\rho_0^2}{2\pi^2} \left(\frac{\partial \tilde{u}_1}{\partial y_1} \right)^2 \int_{\tilde{S}} d\tilde{S}^3 \left[\left(\frac{1}{\tilde{S}} - \cos^2 \beta \frac{S_1^2}{S^3} - \sin \beta \frac{S_2^2}{S^3} \right) \left(\sin^2 \beta \overline{u_1 u_1} + \cos^2 \beta \overline{u_2 u_2} \right) + \right.$$

$$\left. + \left(2 \sin \beta \cos \beta \frac{S_1 S_2}{S^3} \right) \left(2 \sin \beta \cos \beta \overline{u_1 u_2} \right) \right] \quad (\text{A-4})$$

The tilde has been dropped for simplicity. Distances and correlations henceforth are understood to be in the axisymmetric frame.

The axisymmetric formalities developed by Batchelor (1946) and Chandrasekhar (1950) allow expression of the correlations in terms of three defining scalars Q_1, Q_2, Q_3 . Goldstein (1973) assuming incompressibility, has derived explicit expressions for the correlations.

$$\overline{u_1 u_1} = \frac{1}{\sigma} \frac{\partial}{\partial \sigma} (\sigma^2 Q_1(\mathcal{J}, \sigma)) \quad (\text{A-5a})$$

$$\overline{u_1 u_2} = \frac{\partial}{\partial \mathcal{J}_1} (\mathcal{J}_2 Q_1(\mathcal{J}, \sigma)) \quad (\text{A-5b})$$

$$\begin{aligned} \overline{u_2 u_2} = & \left(\mathcal{J}_3 \frac{\partial}{\partial \mathcal{J}_1} - 2 \frac{\partial^2 \mathcal{J}_1 \mathcal{J}_2}{\partial \mathcal{J}_1 \partial \mathcal{J}_2} + \mathcal{J}_1^2 \frac{\partial^2}{\partial \mathcal{J}_3^2} \right) Q_1(\mathcal{J}, \sigma) - \\ & - \frac{\partial}{\partial \mathcal{J}_3} (\mathcal{J}_3 Q_2(\mathcal{J}, \sigma)) \end{aligned} \quad (\text{A-5c})$$

where $\sigma = \sqrt{\mathcal{J}_2^2 + \mathcal{J}_3^2}$

The Q s are the axisymmetric analog of isotropic f functions. Unlike the isotropic f functions, however, the Q functions cannot be readily identified as directly measurable functions and are further complicated by being a function of two variables.

The following special forms are assumed for the defining scalars.

$$Q_1(\mathcal{S}_1, \sigma) = \frac{U_1^2}{2} \exp\left(-\left(\frac{\sigma}{l_2}\right)^2 + \left(\frac{\mathcal{S}_1}{l_1}\right)^2\right) \quad (\text{A-6a})$$

$$Q_2(\mathcal{S}_1, \sigma) = -(\overline{U_2^2} - \overline{U_1^2}) \exp\left(-\left(\frac{\sigma}{l_2}\right)^2 + \left(\frac{\mathcal{S}_1}{l_1}\right)^2\right) \quad (\text{A-6b})$$

These forms were used successfully by Goldstein to predict far field sound generated by mixing layer turbulence. It is easily seen that for an isotropic reduction ($\overline{U_2^2} \rightarrow \overline{U_1^2}; l_2 \rightarrow l_1$) the axisymmetric forms reduce to isotropic forms with $f = \overline{U^2} e^{-\mathcal{S}/l}$. This provides a useful comparison and check. The constants l_1 and l_2 in Eq. A-6a and A-6b can be determined directly by measurements along the $\mathcal{S}_1$ and $\mathcal{S}_2$ axis.

$$\overline{U_1 U_1}(\mathcal{S}_1) = \overline{U_1^2} \exp(-\mathcal{S}_1/l_1) \quad (\text{A-7a})$$

$$\overline{U_2 U_2}(\mathcal{S}_2) = \overline{U_2^2} \exp(-\mathcal{S}_2/l_2) \quad (\text{A-7b})$$

Equations A-5 and A-6 imply the correlations required for integration of Eq. A-4. Defining $R^2 = \left(\frac{\sigma}{l_2}\right)^2 + \left(\frac{\mathcal{S}_1}{l_1}\right)^2$, they are given by

$$\overline{U_1 U_1} = \frac{U_1^2}{2} e^{-R} \left(\frac{\sigma^2}{l_2^2 R} - 2 \right) \quad (\text{A-8a})$$

$$\overline{U_1 U_2} = \frac{U_1^2}{2} e^{-R} \frac{S_1 S_2}{l_1^2 R} \quad (\text{A-8b})$$

$$\begin{aligned} \overline{U_2 U_2} = \frac{\overline{U_2^2}}{2} e^{-R} & \left[\frac{S_1^2 S_3^2}{l_1^4 R^2} - \frac{S_1^2 S_3^2}{l_1^4 R^3} + \frac{S_3^2}{l_1^2 R} + 2 \frac{S_1^2 S_3^2}{l_1^2 l_2^2 R^3} \right. \\ & + 2 \frac{S_1^2 S_3^2}{l_1^2 l_2^2 R^2} - 2 \frac{S_1^2}{l_1^2 R} + \frac{S_1^2}{l_2^2 R} \\ & \left. - \frac{S_1^2 S_3^2}{l_1^4 R^3} - \frac{S_1^2 S_3^2}{l_2^4 R^2} + 2 \frac{\overline{U_2^2}}{U_1^2} \left(1 - \frac{S_3^2}{l_2^2 R} \right) \right] \quad (\text{A-8c}) \end{aligned}$$

A transformation into spherical coordinates is convenient for integrating Eq. A-4. The transformation is

$$S_1 = l_1 R \cos \varphi \quad (\text{A-9a})$$

$$S_2 = l_2 R \sin \varphi \cos \theta \quad (\text{A-9b})$$

$$S_3 = l_2 R \sin \varphi \sin \theta \quad (\text{A-9c})$$

With this transformation, we make use of

$$\left(\frac{\sigma}{l_2}\right)^2 + \left(\frac{\rho_1}{l_1}\right)^2 = R^2 \quad (\text{A-10a})$$

$$\int_{\vec{s}} d\vec{s}^3 = l_1 l_2^2 \int_0^\infty dR R^2 \int_0^\pi d\varphi \sin\varphi \int_0^{2\pi} d\Theta \quad (\text{A-10b})$$

$$s^2 = R^2 l_2^2 \left[\left(\left(\frac{l_1}{l_2} \right)^2 - 1 \right) \cos^2 \varphi + 1 \right] \quad (\text{A-10c})$$

For simplicity, the scale and intensity anisotropy ratio is defined by

$$\alpha = l_1/l_2 \quad (\text{A-11})$$

$$k = \left(\frac{\overline{U_2^2}}{\overline{U_1^2}} \right)^{1/2} \quad (\text{A-12})$$

After the spherical transformation of Eqs. A-4 and A-8, Eqs. A-8 are substituted into Eqs. A-4. The integrations are carried out over Θ, R and finally φ . The result is

$$P_{TM}^2 = \varphi^2 \left(\frac{\partial \overline{U_1}}{\partial y_2} \right)^2 \overline{U_1^2} l_1 l_2 I(\alpha, \beta, k) \quad (\text{A-13})$$

where $I(\alpha, \beta, k)$, the analog of Kraichnan's $J(\alpha)$, is a function of the scale anisotropy α , intensity anisotropy k , and axisymmetric orientation β . In recursive form, $I(\alpha, \beta, k)$ is given by

$$I = \left[\begin{array}{l} \left\{ \begin{array}{l} 2 \sin^2 \beta (I_2) - 2 \sin^2 \beta \cos^2 \beta \alpha^2 (I_6) \\ - \sin^2 \beta (I_5 - I_6) \\ \cos^2 \beta [4(2 - \alpha^2 - \frac{1}{2} \alpha^2)(I_2 - I_3) + 2(\alpha^2 - 2)I_2 + \\ \frac{1}{2} \alpha^2 (I_1 - I_2) + 2k^2 I_2] \\ - \cos^4 \beta \alpha^2 [4(2 - \alpha^2 - \frac{1}{2} \alpha^2)(I_6 - I_7) + 2(\alpha^2 - 2)I_6 \\ + \frac{1}{2} \alpha^2 (I_5 - I_6) + 2k^2 I_6] \\ - \sin \beta \cos^2 \beta [(2 - \alpha^2 - \frac{1}{2} \alpha^2)(I_5 + 2I_6 + I_7) + (\alpha^2 - 2)(I_5 - I_6) \\ + \frac{1}{4} \alpha^2 (I_4 - 2I_5 + I_6) + \frac{k^2}{2} (I_4 - I_6) \\ + 4 \sin^2 \beta \cos^2 \beta (I_5 - I_6)] \end{array} \right\} ; \alpha \neq 1 \\ \\ \left\{ \begin{array}{l} \frac{4}{3} \sin^2 \beta - \frac{4}{5} \sin^2 \beta \cos^2 \beta - \frac{4}{5} \sin^3 \beta \\ + \frac{4}{3} k^2 \cos^2 \beta + (\frac{8}{5} - \frac{4}{5} k^2) \cos^4 \beta \\ - \frac{4}{5} k^2 \sin \beta \cos^3 \beta + \frac{16}{5} \sin^2 \beta \cos^2 \beta \end{array} \right\} ; \alpha = 1 \end{array} \right] \quad (A-14)$$

where

$$I_1 = \begin{cases} 2/(\alpha^2 - 1)^{1/2} \cosh^{-1}(\alpha) ; \alpha > 1 \\ 2/(\alpha^2 - 1)^{1/2} \cos^{-1}(\alpha) ; \alpha < 1 \end{cases} \quad (A-15a)$$

$$I_2 = \frac{\alpha}{\alpha^2 - 1} - \frac{1}{2(\alpha^2 - 1)} I_1 \quad (A-15b)$$

$$I_3 = \frac{1}{2} \left(\frac{\alpha}{\alpha^2 - 1} \right) - \frac{3}{4} \frac{\alpha}{(\alpha^2 - 1)^2} + \frac{3}{8} \frac{1}{(\alpha^2 - 1)^2} I_1 \quad (A-15c)$$

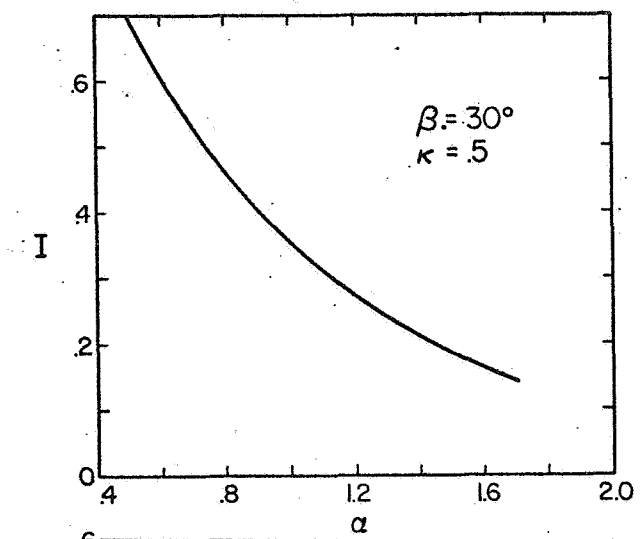
$$I_4 = \frac{2}{\alpha} \quad (\text{A-15d})$$

$$I_5 = \frac{-2}{(\alpha^2-1)^2} + \frac{1}{(\alpha^2-1)} I_1 \quad (\text{A-15e})$$

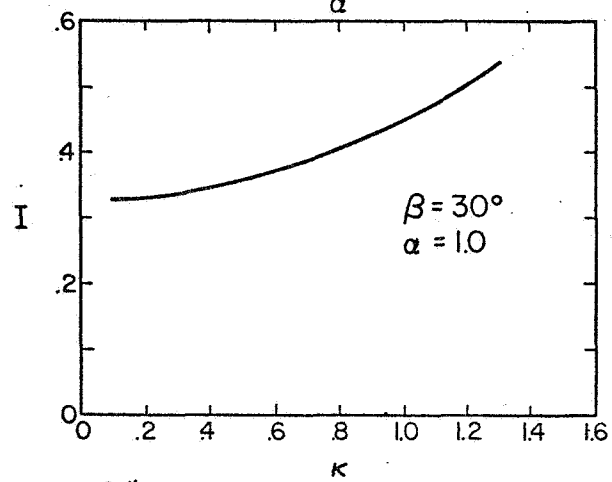
$$I_6 = \frac{\alpha}{(\alpha^2-1)^2} + \frac{2}{\alpha(\alpha^2-1)^2} - \frac{3}{2(\alpha^2-1)^2} I_1 \quad (\text{A-15f})$$

$$I_7 = \frac{1}{2\alpha(\alpha^2-1)} - \frac{5}{4\alpha(\alpha^2-1)^2} - \frac{15}{4\alpha(\alpha^2-1)^3} + \frac{15}{8(\alpha^2-1)^3} I_1 \quad (\text{A-15g})$$

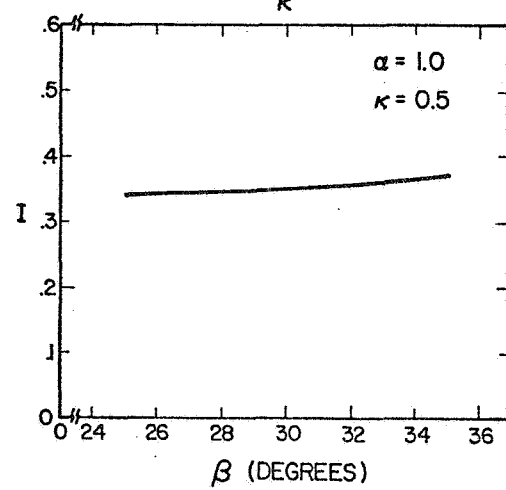
A consistency check shows the axisymmetric prediction degenerates into the isotropic prediction. Specifically as $\alpha \rightarrow 1.0$, $k \rightarrow 1.0$, $\beta \rightarrow 0.0$ then $I(\alpha, \beta, k) \rightarrow 8/15$, which is Kraichnan's isotropic result for $f = e^{-8/15}$. Typical values of $I(\alpha, \beta, k)$ are plotted in Fig. A-1 for parameters expected in a free mixing layer. Because the turbulent-mean shear pressure is generated by turbulent velocities and mean velocity gradients normal to the flow direction, a jet coordinate analysis, rather than an axisymmetric system analysis, gives a clearer indication of the effects of turbulent structure. However, the trends shown in Fig. A-1 are as expected. Referring to Eq. A-13 and Fig. A-1a, we see, for constant correlation area $(l_1 l_2)$, decreasing α corresponds to decreasing l_2 , which is the correlation length most directly across the shear layer. This, agreeing with the work in Chapter 2, decreases



(a)



(b)



(c)

Fig. A-1 Anisotropy Factor for Axisymmetric Turbulence.

P_{TM}^2 Figure A-1b shows, with all else constant, increasing the intensity across the shear layer increases P_{TM}^2 . The slow variation in A-1c shows that near $k = .5$ and $\alpha = 1.0$, P_{TM}^2 is not strongly dependent on the axisymmetric orientation. The variation existing is a result of increased intensity across the shear layer.

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